Litland's "Grounding and defining identity"

1. Claims

(Null$_=$) $\Box_= \forall x \forall y(x = y \rightarrow ((0 \ll x = y) \wedge \forall pp(pp \ll x = y \rightarrow 0 \equiv pp)))$

- "It is essential to identity that the only ground of identity facts is 0."

(Null$_\neq$) $\Box_= \forall x \forall y(x \neq y \rightarrow ((0 \ll x \neq y) \wedge \forall pp(pp \ll x \neq y \rightarrow 0 \equiv pp)))$

- "It is essential to identity that the only ground of distinctness facts is 0."

(i) Litland is operating with a notion of strict, full, immediate ground.
(ii) Litland holds a parallel thesis for the identity relation of each type $< \tau, \tau >$.
(iii) Litland allows the essentialist operator to be indexed with items of arbitrary types.

2. Argumentative strategy

Step 1: there is a relation R such that (Null$_R$) and (Null$_{\lambda xy.\neg Rxy}$) follows from the real definition of R.

Step 2: R is the best candidate of =.

3. Real definition

A rough idea: to define a relation, one specifies the grounds of its instantiations and non-instantiations.

<u>3.1. Grammar</u>

Litland works in the language of simple, relational type theory, which contains types for pluralities.

In general, a claim of (full, immediate) real definition can be expressed as

$$s \xrightarrow[x]{x_0 : \psi_0, x_1 : \psi_1, \ldots} \Vert \phi_0, \phi_1, \ldots$$

which reads: s is, by definition the x such that for all x_0 that satisfies ψ_0, x_1 that satisfies ψ_1,…, $\phi_0(x, x_0, x_1, \ldots)$, $\phi_1(x, x_0, x_1, \ldots)$,…

Litland includes as a primitive the application relation $\mathscr{A}(Rxyp)$, which reads "p is a result of applying R to x and y.

Here, the "is by definition" connective plays a complex grammatical role: it binds and generalizes variables, and restricts the range of variables it generalizes.

This complex grammar is mainly motivated by the idea that *logical operations* should be *pure*. Litland characterizes the intuitive notion of purity as

- A definition is immediately (¬-)pure if the only items figuring in it are the grounding relation and the application relation (and negation).
- An item is immediately (¬-)pure if its definition is immediately (¬-)pure.
- An item is (¬-)pure if it is either immediately (¬-)pure or has a definition from (¬-)pure items.

To satisfy the requirement that identity be (¬-)pure, its definition can't have quantifiers, conditionals, etc. in it, hence the complex grammar of the definition connective.

"Wittegensteinian variable convention": distinct bound variables take distinct values.

3.2. Logic

Factivity: if s is by definition the x such that $\phi(x)$, then $\phi(s/x)$.
Individuation: no two items have the same real definition.
Definitional $\ll$-reflection: if p is, by definition, (anti-)grounded in qq, then qq are the only (immediate non-factive) (anti-)grounds of p.
Definitional $\mathscr{A}$-reflection: if p is, by definition, a result of applying R to x and y, then p can only results from applying R to x and y.

3.3. Metalogic

- A plurality pp of propositions is *closed* iff
  - (i) pp contains the definition of every item figuring in any proposition in p;
  - (ii) pp is closed under Factivity, Individuation, $\ll$-reflection and $\mathscr{A}$-reflection;
  - (iii) pp contains every proposition that is grounded in some propositions amongst the pp; and
  - (iv) any proposition amongst the pp that has some full grounds, has some full grounds amongst the pp.

- pp is *coherently closed* if it is closed doesn't contain a contradiction.
- q is a *consequence* of pp if q is in every closed plurality containing pp.
- q is a *coherent consequence* of pp if q is in every coherently closed plurality containing pp.

- q is a *essential coherent* consequence of pp if q is a coherent consequence of pp, and every item that figures in q figures in a definition of an item in pp.

4. Really defining identity

$$= \underset{R}{\overset{x,y,(p:\mathcal{A}(Rxxp)),(q:\mathcal{A}(Rxyq))}{\rule{0pt}{0pt}\overline{\qquad\qquad\qquad}}}\!\dashv\!\| \begin{cases} p \underset{r}{\dashv\|} \begin{cases} \mathcal{A}(Rxxr) \\ 0 \ll r \end{cases} \\ q \underset{r}{\dashv\|} \begin{cases} \mathcal{A}(Rxyr) \\ \mathcal{A}(Ryxr) \\ 0 \ll \neg r \end{cases} \end{cases}$$

which roughly reads: identity is, by definition, the relation R such that for all x and y, for all p resulting from applying R to x and x, and for all q resulting from applying R to x and y,
(i) p is, by definition, a result of applying R to x and x, and is 0-grounded;
(ii) q is, by definition, a result of applying R to x and y, and is 0-grounded.

Leibniz's Law, Strict Symmetry, and the claim that identity and distinctness facts are uniquely 0-grounded, follows from this definition and the logic sketched in 3.2.

Litland also sketches an argument to the effect that

$$\forall x \forall y(x = y \rightarrow ((0 \ll x = y) \wedge \forall pp(pp \ll x = y \rightarrow 0 \equiv pp)))$$

is an essential consequence of the real definitions of the logical operations involved. Presumably, given certain principles connecting claims involving $\Box_x$ and claims about essential consequence that he doesn't specify, one could deduce ($\text{Null}_=$) and ($\text{Null}_{\neq}$).

5. Theoretical virtues

Litland carries out Step 2 of his argumentative strategy by arguing (1) that his definition of identity validates standard logical principles involving identity; and (2) that his account has several theoretical virtues that his competitors lack.

In particular, he argues that his account
- (i) is uniform and topic-neutral;
- (ii) avoids problems that competitors have;
- (iii) avoids the objection from differential grounding,