Symmetric relations, symmetric theories, and Pythagrapheanism

The Orthodox: It's not the case that every basic relation is symmetric. Specifically, no interesting non-symmetric relations non-symmetric relations are reducible to symmetric ones.

Non-interesting relations: relations like $R(x, y)$:= iff $F(x) \wedge \neg F(y)$. R is not symmetric but easily reducible.

Aim: There is no compelling reason to insist that there must be non-symmetric basic relations.

Assume that Gottftied defines P_R^x as $\lambda y R(x, y)$, and proposes the reduction from $R(x, y)$ to P_R^x. Something is wrong: first-order logic is decidable, but polyadic first-order logic is undecidable. So no reduction.

In order to overthrow the orthodoxy, we must (at least) show how we can theorize about non-symmetric relations using only symmetric relations.

A predicate, R, is symmetric in a theory T iff both R is two-placed and $T \vdash \forall x \forall y (R(x, y) \rightarrow R(y, x))$; otherwise, R is non-symmetric in T. The theory T itself is symmetric iff every T-primitive is symmetric in T.

A theory, T, is a graph theory iff Tfis only non-logical primitive is “E”, which is symmetric and irreflexive in T, i.e. $T \vdash \forall x \neg E(x, x)$.

A map $H : \Sigma \to \Sigma'$ is a reconstrual if it satisfies the following condition:

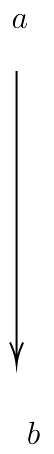
1. For every n-ary predicate $P \in \Sigma$, $HP(x_1, \dots, x_n)$ is a Σ'-formula with n free variables.
2. For every constant $c \in \Sigma$, Hc is a c.

A map $*$ from Σ-formulae to Σ'-formulae is a translation (relative to H) from Σ to Σ' if $*$ satisfies the following conditions:

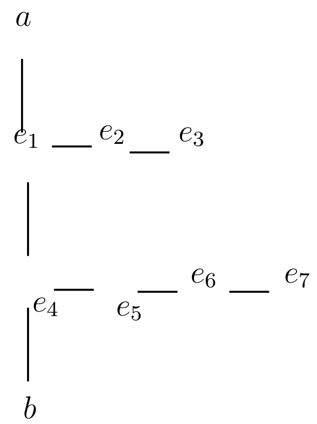
1. $(Pt_1 \dots t_n)^* := \exists x_1 \dots x_n(ht_1(x_1) \wedge \dots \wedge ht_n(x_n) \wedge HP(x_1, \dots, x_n))$,
2. $(\exists x \phi(x))^* := \exists x(\delta_\eta(x) \to \phi^*)$, where $\delta_\eta(x)$ is a Σ'-formula called domain formula,
3. η commutes with Boolean connectives.

An interpretation $* : T \to S$ is translation such that if $T \vdash \phi$ then $S \vdash \phi^*$. $*$ is faithful iff $T \vdash \phi$ iff $S \vdash \phi^*$.

1 The Rough Idea



R_D



R_G

A non-symmetric relation can be regarded as a directed graph R_D. R_G is a way using undirected graph to reconstruct R_D: let E be R_G's edge relation. Define $Old(x) :=$ $\forall v(E(x, v) \to$ (exactly 3 entities have edges to v)). Define $R^* :=$ there are $e_1, \ldots, e_7$ such that: $E(x, e_1), E(y, e_4), E(e_1, e_2), \ldots, E(e_6, e_7)$, but there are no other edges involving any of $e_1, \ldots, e_7$.

Now we can define a translation $*$: where ϕ is a first-order formula whose only non-logical primitive is R, let ϕ^* be the result of first restricting all of ϕ's quantifiers to Old, and then replacing any subformula of the form $R(x, y)$ with $R^*(x, y)$. Then for any old nodes $a_1, \ldots, a_n$, $R_D \models \phi(a_1, \ldots, a_n)$ iff $R_G \models \phi^*(a_1, \ldots, a_n)$.

This suggests a method for reducing the non-symmetric relation, R, to a symmetric relation: claim thatR_G' edge relation, E, is more basic than R, and that R is perspicuously analysed via R^*.

Generalize: Suppose that Tfis only primitive is R. Then, with $*$ defined as above, let

T_{new} be the graph theory whose axioms are exactly ϕ^*, for any T-axiom ϕ, plus an extra axiom which ensures that T_{new} is a graph theory, i.e. "E is symmetric and irreflexive". It is now easy to show that $*$ is a faithful interpretation, in that: $T \vdash \phi$ iff $T_{new} \vdash \phi^*$, for any T-sentence ϕ. In other words, $*$ is a faithful interpretation.

2 The Problem

The problem with the above strategy is that some R is unrestricted in the R_D sense, but is restricted in the R_G sense.

Theories T and T' are synonymous iff there are interpretations $\eta : T \to T'$ and $\beta : T' \to T$ such that $T \vdash \phi \leftrightarrow \beta\eta\phi$ and $T' \vdash \psi \leftrightarrow \beta\eta\psi$ for every T-formula ϕ and every S-formula ψ.

The theory of T and T_{new} defined previously may not be synonymous. The problem we considered can be paraphrased as: T_{new} interprets R as a restricted relation, R^*. The orthodox should be: no interesting theory is synonymous with any symmetric theory. Or, every interesting theory is unsymmetrizable.

A theory, T, is unsymmetrizable iff no symmetric theory is synonymous with T.

3 The Precise Idea

A theory, T, is graphable iff T is synonymous with some graph theory.

Button's refutation of the orthodoxy really comes down to this point: Vast swathes of mathematical theories are graphable.

Proposition 1. *Let T be a first-order theory, with finitely many primitives, which directly interprets AS_e. Then T is graphable.*

Note that Proposition 1 shows the following: Take any theory on the list just given, or just start with AS_e itself. Next, enrich your chosen theory with some first-order axiomsfi?!as many as you like. If you want, you may formulate these axioms using new primitives, provided that you use only finitely many new primitives. Now: whatever you did, the resulting theory is graphable.

4 Pythagrapheanism

Assumption 1: Our favourite physical theory can be formulated so that it directly interprets AS_e.

Assumption 2: Our favourite physical theory uses only finitely many non-logical primitives.

The conclusion is that our favourite physical theory is graphable. So, our favourite physical theory is reducible to a graph theory. In sum: there is no formal impediment to the claim that you, me, and everyone we know are all just nodes in an enormous graph, and that all the various non-symmetric relations— Love, Hate, and everything else— reduce to that graphfis edge relation. Otherwise put: the orthodoxy is so wrong, that perhaps every relation reduces to a single, symmetric relation.

Limitation of this method: The construction of a graph theory T_{graph} of a non-symmertical theory T involves certain arbitrary technical choices. There could be some equally good graph theories like T_{alt} as an equally good candidate for being the fundamental theory.