Schematic Generality

For a sentence like ‘Everything is self-identical’, one may express it quantificationally:

$$\forall_x (x = x)$$

which requires a domain of quantification, which is mostly a set, that includes absolutely everything.

Problem: a set of absolutely everything may face paradoxes.

e.g., the set of absolutely everything should contain the set of all non-self-membered sets, i.e., the Russell Set.

However, the Russell Set is paradoxical: the set is self-membered if and only if it is not self-membered.

Alternative: schemas and schematic generality.

The structure of schemas: a schematic formula to be substituted by instances and some side-condition(s) specifying what an admissible instance must satisfy.

Example: ‘Everything is self-identical’ understood schematically in terms of a schematic formula: $s = s$, where s is a schematic letter and something like *being a thing* is the side-condition such that what’s allowed for substituting the schematic letter must be a thing.

Schematic formulas express a kind of commitment to the truth of what’s a substitution instance of a formula, so although they are not truth-bearers, they are related to truth.

Feature: (1) open-endedness; (2) doesn’t rely on quantificational generality

(1) the open-endedness of schemas amounts to their allowing for new substitution instances as the language in use expands. (So, when a new thing comes into existence (with a new name attached to it), that everything is self-identical requires that thing’s self-identity)

(2) schematic generality doesn’t require a domain of quantification, but rather that something can replace the schematic variable so long as it satisfies the side-conditions.

To differentiate, say ‘*Everything* is self-identical’ triggers a quantificational understanding, while ‘*Anything* is self-identical’ triggers a schematic understanding.

Two problems for schematic generality (as presented by James Studd**)**:

a) schematic formulas cannot be negated as desired. When negating a general claim, we usually expect an existential claim. (e.g., ‘I don’t know every poem’ means that there is some poem that I don’t read) But for ‘I don’t know any poem’, schematically expressed as $\neg Know(p)$, since schemas express a sort of commitment to truth, the negative schema expresses a commitment to the falsity of substituting instances, such that there is no poem that I know.

b) schematic formulas still require an understanding of quantificational generality. To understand what counts as an admissible instance is to be able to distinguish the instances from non-instances. And that in turn requires an understanding of *all* admissible instances, which is quantificational.

Any Solution?

The semantics of the English word ‘any’ might help.

Russell’s analysis:

> [T]he triangle taken is *any* triangle, not some one special triangle; and thus although, throughout the proof, only one triangle is dealt with, yet the proof retains its generality. If we say: ‘Let ABC be a triangle, then the sides AB, AC are together greater than the side BC’, we are saying something about *one* triangle, not about *all* triangles; but the one triangle concerned is absolutely ambiguous, and our statement consequently is also absolutely ambiguous. We do not affirm any one definite proposition, but an undetermined one of all the propositions resulting from supposing ABC to be this or that triangle. (Russell 1908 (“Mathematical Logic as Based on the Theory of Types”), p. 227, Russell’s emphases)

‘Any’ expresses what he calls *systematic ambiguity*, such that, for example, when a speaker says, ‘Let ABC be *any* triangle’, the identity of ABC is unspecified, or systematically ambiguous. And since it is unspecified with respect to the identity of ABC, what’s true of ABC is true in general of triangles.

Dieveney's analysis: 'any' expresses open arbitrary generality. The openness is the same as the open-endedness of schemas, that the range of admissible substitution instances is not fixed, and expands as the language in use expands. The arbitrariness amounts to the way of picking out the object in question, say a triangle, such that which triangle is actually selected is undetermined and the only thing certain is that it's a triangle. (Dieveney, p. 126)

Summarizing: 'any' expresses a kind of linguistic unspecificity, such that the identity of the object in question is not specified except that the object satisfies the relevant side-conditions. Example: When I say, 'Any book from the shelf can be picked', although some specific book will be picked at a time, it doesn't matter which book it is, as long as it is a book on the shelf, hence generality is expressed. (similar to how the universal introduction rule works in standard logic textbook)

'Any' and Schemas Combined: schematic generality comes from the linguistic unspecificity representative in statements using 'any', such that a schematic formula expresses the commitment to the truth of *any* admissible instance.

Saving Schemas:

Responding to objection b) from the side-conditions, there is no need to distinguish instances from non-instances through quantifying over a domain of *all* admissible instances. Instead, we may say that the side-condition holds for *any* admissible instance, expressing a commitment to the truth of something that is an instance satisfying the side-condition, while the identity of the admissible instance talked about is unspecified. Since the identity is unspecified, what holds of this admissible instance holds generally for something so long as it is an admissible instance.

Responding to objection a) from negations, it is important to keep in mind that schemas express a sort of commitment and are thus not truth-bearers. While negation is a truth-functional operator, it is thus incorrect to attempt to negate non-truth-bearers. So, instead of directly negating a schematic formula, one may instead negate the expressed commitment to truth. Negating a commitment to truth of admissible instances means admitting some admissible instances for which the relevant property fails to hold. (e.g., negating the commitment expressed by that any even number is divisible by two means admitting some even numbers that are not divisible by two)

Further Problem: making a commitment is like having some kind of belief yet believing that self-identity fails to hold for something doesn't mean the expected existential claim that something really is not self-identical. Similarly, that the identity of any triangle ABC is unspecified doesn't mean that ABC is metaphysically unspecified.

Answer: at least when making absolutely general claims, like those axioms in set theory, they have a similar status as the schemas, such that they are accepted as the basis to prove theorems and other truths, and such an acceptance is similar to commitment.