# Ontology and Arbitrariness

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## 1 Introduction

- Considerations of arbitrariness only seem to push us towards opposite extremes.
- It seems that one must appeal to additional theoretical desiderata to decide between maximalist views an minimalist view.
- It would be nice if anti-arbitrariness reasoning could help us deciding between the two opposite extremes.
- My main goal is to show that an argument for minimalism over maximalism can be given, purely on the grounds of anti-arbitrariness.

## 2 Indefinite Extensibility and Arbitrariness

**Collapse**: $\forall xx \exists y (Set(y) \wedge \forall x (x \prec xx \leftrightarrow x \in y))$ (from Linnebo[2010,2013])

- Non-arbitrariness.
- The semantics of plural quantification ensures that it is determinate which things are among xx, and a set is completely characterized by specifying its elements. We can thus give a complete and precise characterization of the set that xx would form if they did form a set. What more could be needed for such a set to exist?
- Inconsistent with plural comprehension. Leads to a Russellian Paradox.
- Inconsistent with the plural version of Cantor's theorem, which entails that the overwhe1ming majority of pluralities do not form sets.To see this, apply the theorem to the plurality of absolutely all objects. than there are objects and thus a foniori more pluralities than there are sets.

**Set Indefinite Extensibility(SIE)**: $\Box \forall xx \diamond \exists y (Set(y) \wedge \forall x (x \prec xx \leftrightarrow x \in y))$

- $\Diamond$ : logically possible and conceivable.
- Non-arbitrariness: Deniers of SIE need to provide an explanation for why it is logically impossible for certain pluralities to form a corresponding set, otherwise their position is objectionably arbitrary.

- SIE is entirely immune to the set-theoretic paradoxes. SIE in no way clashes with the purely mathematical fact that there is no set of all sets. The existence of such a set would directly contradict the axiom of regularity, since the set of all sets would have to contain itself. However, this mathematical fact is entirely irrelevant to the claim that for any things, it is logically possible or conceivable that they form a set(?).
- Deniers of SIE need to provide an explanation for why it is logically impossible for certain pluralities to form a corresponding set, otherwise their position is objectionably arbitrary.

**Cardinal Indefinite Extensibility(CIE)**: $\Box\forall xx(\forall x(x \prec xx \rightarrow Cardinal(x)) \rightarrow \Diamond\exists y(Cardinal(y)\wedge \forall x(x \prec xx \rightarrow y > x))$

## 3 The argument against maximalism of mathematical objects

If there is a plenitude of all possible mathematical objects, then there is a plenitude of all possible pure sets. Informally, there is a plenitude of pure sets just in case there couldn't be more pure sets than there actually are. Therefore we have:

**Set Plenitude**: $\exists xx(\forall x(x \prec xx \rightarrow Set(x)) \wedge \neg \Diamond \exists y(Set(y) \wedge \forall x(x \prec xx \rightarrow y \not\equiv x))$

**The argument** goes as follows:
1. If maximalism is true, Set Plenitude is true.
2. SIE is true.
3. If SIE is true, Set Plenitude is false.(Take the plurality of all pure sets that there are)
4. Therefore, maximalism is false.

## 4 The argument against maximalism of other objects

**The schematic argument**:
1. If maximalism is true, ... Plenitude is true.
2. SIE and CIE is true.
3. If SIE and CIE is true, ... Plenitude is false.
4. Therefore, maximalism is false.

Instances of ... Plentitude:

- **Proposition Plenitude**:If it is possible that there be some cardinal number $\kappa$, then there is the proposition that there are exactly $\kappa$-many angels.
- **Property Plenitude**:Property Plenitude: If it is possible that there be some cardinal number $\kappa$, then there is the property of being in a world with at most $\kappa$ angels.

- **Modal Plenitude**:If it is possible that there be some cardinal number $\kappa$, it is possible that there are exactly $\kappa$-many angels. (Diffrent reading of modal operator?)

## 5 Possible reply

- Lewis's Intermediate Realism
- Insisting that the only intelligible reading of the modal operators in SIE corresponds to metaphysical modality, while at the same time adopting the view that the ontological truths of metaphysics are metaphysically necessary.
- Questioning the possibility of quantifying over absolutely everything that there is, which was essential to the arguments that I presented.