Logics of Synonymy

May 14, 2022

The paper studies a number of logics of synonymy. The author suggests four theses that a notion of synonymy is supposed to respect. However, he argues that no account of synonymy under study can satisfy all. He considers ways to relax some of these constraints and arguments for different notions of synonymy.

1 Preliminary

Hyperintensionality, the granularity problem, and logics of synonymy. Note: The author intends to investigate the notion of synonymy without any commitment to a particular semantic framework. So he starts with logics that attempt to capture the notion.

The paper is intersted in the *logical* laws of synonymy, rather than when atmoic sentences are synonymous. The author supposes that synonymous atomic sentences are always assigned a unique atomic symbol.

Remarks on **Subject Matter**:

1. A *strong* conception of synonymy: Synonymy entails subject matter identity.
2. *Completeness of syntactic reflection*: If two sentences have the same subject matter, they have the same atomic sentences

Four fundamental principles of synonymy:

(P1) *Subject matter preserving*: If two sentences are synonymous, then the sets of their atomic sentences are identical.

(A1) Scenarios are structures where atomic sentences can be evaluated with truth-values in $\{t, f, u\}$, $\{t, f, b\}$, or $\{t, f, u, b\}$.

(A2) The *truth-value* of a complex sentence at a scenario is determined by the *truth-value* of the sentences parts at that scenario (according to the truth-functions for the connectives as in K3, LP, or FDE, respectively).

(P2) *Scenario respecting*: If there is no possible scenario or circumstance whatsoever in which two sentences differ in truth-value, then they are synonymous.

One observation: any scenario-based account that satisfies (A1) and (A2) cannot satisfy (P1). Consider p and $p \vee (p \wedge q)$.

2 Systems

AC in Fine (2016): AC $\vdash \varphi \equiv \psi$ iff they have the same replete content (See my note on "Truthmaker Equivalence").

(A1)	$\varphi \equiv \neg\neg\varphi$	(A7)	$(\varphi \vee \psi) \vee \chi \equiv \varphi \vee (\psi \vee \chi)$
(A2)	$\varphi \equiv \varphi \wedge \varphi$	(A8)	$\neg(\varphi \wedge \psi) \equiv (\neg\varphi \vee \neg\psi)$
(A3)	$\varphi \wedge \psi \equiv \psi \wedge \varphi$	(A9)	$\neg(\varphi \vee \psi) \equiv (\neg\varphi \wedge \neg\psi)$
(A4)	$(\varphi \wedge \psi) \wedge \chi \equiv \varphi \wedge (\psi \wedge \chi)$	(A10)	$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi)$
(A5)	$\varphi \equiv \varphi \vee \varphi$	(A11)	$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi)$
(A6)	$\varphi \vee \psi \equiv \psi \vee \varphi$		

(R1) $$\frac{\varphi \equiv \psi}{\psi \equiv \varphi}$$

(R2) $$\frac{\varphi \equiv \psi \qquad \psi \equiv \chi}{\varphi \equiv \chi}$$

(R3) $$\frac{\varphi \equiv \psi}{\varphi \wedge \chi \equiv \psi \wedge \chi}$$

(R4) $$\frac{\varphi \equiv \psi}{\varphi \vee \chi \equiv \psi \vee \chi}$$

Figure 1: The system of analytic containment

Note that the truthmaking conception of synonymy does not satisfy (A2) above: note that $s \Vdash \varphi \wedge \psi$ iff $\exists u, t$ s.t. $s = u \sqcup t$ and $u \Vdash \varphi$ and $t \Vdash \psi$. So to determine whether a conjunction is (exactly) verified by a state, we may need to look at some other states.

Note that AC is not the minimal logic of truthmaker semantics. See "Truthmaker Equivalence" and my note.

The logic of scenario-based synonymy (FDE) SF = AC $\oplus\, \varphi \vee (\varphi \wedge \psi) \equiv \varphi$.

Truthmaking synonymy and scenario synonymy are related by moving up one set-theoretic level: moving from scenarios to sets of scenarios (as the entities at which sentences are evaluated) fine-grains scenario semantics to the level of truthmaking semantics.

Let $L(\varphi) = \{p : p \text{ occurs positively in } \varphi\} \cup \{\neg p : p \text{ occurs negatively in } \varphi\}$.

Definition 3 The logics that we'll consider are axiomatized as follows.

$$\begin{aligned}
\mathsf{SFL} &:= \mathsf{AC} \\
\mathsf{SF} &:= \mathsf{AC} + \varphi \vee (\varphi \wedge \psi) \equiv \varphi \\
\mathsf{SFA} &:= \mathsf{AC} + \varphi \vee (\varphi \wedge \psi) \equiv \varphi \vee (\varphi \wedge \neg\psi) \\
\mathsf{SCL} &:= \mathsf{AC} + \varphi \equiv \varphi \vee (p \wedge \neg p) && \text{if } p, \neg p \in L(\varphi) \\
\mathsf{SCA} &:= \mathsf{AC} + \varphi \equiv \varphi \vee (p \wedge \neg p) && \text{if } p \in At(\varphi)
\end{aligned}$$

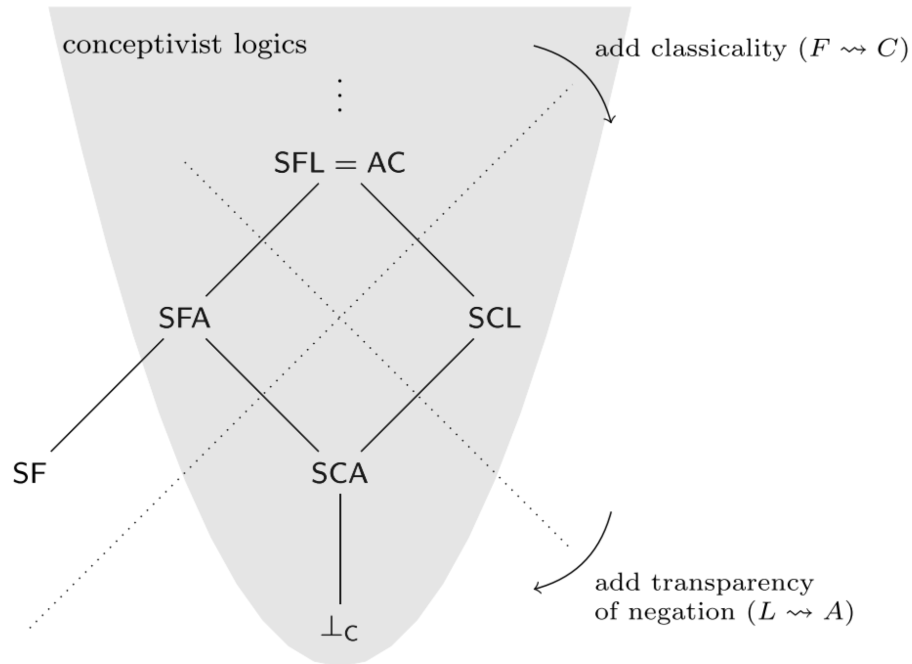

Fig. 2 The discussed benchmark logics of synonymy and the lattice of conceptivist logics

Theorem 2 (**Characterization theorem**) *We have for all sentences φ and ψ that*

(i)	$\mathsf{SF} \vdash \varphi \equiv \psi$	*iff*	$\varphi \Leftrightarrow_{\mathsf{FDE}} \psi$		
(ii)	$\mathsf{AC} = \mathsf{SFL} \vdash \varphi \equiv \psi$	*iff*	$\varphi \Leftrightarrow_{\mathsf{FDE}} \psi$	*and*	$L(\varphi) = L(\psi)$
(iii)	$\mathsf{SFA} \vdash \varphi \equiv \psi$	*iff*	$\varphi \Leftrightarrow_{\mathsf{FDE}} \psi$	*and*	$At(\varphi) = At(\psi)$
(iv)	$\mathsf{SCL} \vdash \varphi \equiv \psi$	*iff*	$\varphi \Leftrightarrow_{\mathsf{C}} \psi$	*and*	$L(\varphi) = L(\psi)$
(v)	$\mathsf{SCA} \vdash \varphi \equiv \psi$	*iff*	$\varphi \Leftrightarrow_{\mathsf{C}} \psi$	*and*	$At(\varphi) = At(\psi)$.

The characterization theorem shows that these notions of synonymy can be determined by a "truth component" and an "aboutness component".

Note that the last three systems do not satisfy the (P2).

3 Arguments for Various Notions of Synonymy

3.1 Argument for SF

Why should we accept $\varphi \equiv \varphi \vee (\varphi \wedge \psi)$. Because this helps us explain Hurford's constraint which says that disjunctions where one disjunct entails the other are infelicitous.

It is a pragmatic principle that you should utter a simpler sentence rather than an equivalent but more complicated one. So to use this principle to explain why a Hurford disjunction is infelicitous, we should accept that such a disjunction is equivalent to one of its disjuncts.

3.2 Argument for SFA

The plausibility of SFA is grounded in the plausibility of scenario-based semantics (FDE) and the *soundness of syntactic reflection*— (S2) If two sentences have the same atomic sentences, they have the same subject matter.

3.3 Argument for AC

Reject (S2)

3.4 Argument for SCA

For:

1. Endorse standard possible world semantics.
2. Kleene logic with the truth-value u interpreted as meaninglessness or off-topicness.

Against: $\varphi \vee (\varphi \wedge \psi)$ should not be equivalent to $(\varphi \wedge \neg\psi) \vee (\varphi \wedge \psi)$. Consider the exhaustive reading of a Hurford disjunction.

3.5 Argument for AC and SCL

Explanational equivalence.

4 Ways Out of the No-go Result

The author intends to relax (P2), (A1), or (A2). To do this requires us to add more structure to a scenario:

1. Add topic: each scenario is accompanied by a set of topic.
2. Consider explanation: truthmaker?
3. Imaginative equivalence.

4.1 Consequences

Pluralism

Non-compositionality If we choose to violate (A2), then the meaning is not *straightforwardly* compositional, in the sense that the *truth-value* of a proposition at a scenario is not determined by the truth-values of its components at the scenario.

Note that truthmaker semantists claim that their semantics is supposed to be a compositional semantics, in the sense that the content of a proposition, i.e., the set of its truthmakers and/or that of its falsitymakers, is determined by the contents of its components.

Positive evidence: logical programming and neural network.

Two ways to relax (A2):

1. Interpreting truth-values as truth-topic composites: extentional truth-functions but intentional truth-values.
2. Endorsing a modal reading of connectives: non-extensional connectives and pure truth-values.

References

Fine, K. (2016). Angellic content. *Journal of Philosophical Logic*, 45(2):199–226.