“Questions in Action” by Daniel Hoek

Handout zur Structura-Lesegruppe

presented by Evan Zhan

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1. Introduction

1.1 The paper has two parts—A: it proposes an inquisitive belief-action principle; B: it applies the principle to motivate a new theory of belief incorporated with a new treatment of deductive inquiry.

For the latter, he not only aims to explain why our beliefs are not deductively closed but also why deduction is still possible and useful, i.e. to address the practical problem of deduction. This is done by exploiting the *role of questions and question-directed beliefs in decision making*.

1.2 Inquisitive agents do not necessarily believe every deductive entailment of what they believe, but they do believe every *part* of what they believe.

Two ways to flesh out this parthood relation:

(i) Propositional Parthood

(ii) Unstructured Content with a novel parthood relation

1.3 *Classical Belief-Action Principle:* A belief that p manifests itself in behavior as a general disposition to act on p.

Inquisitive Belief-Action Principle: A belief that A^Q manifests itself in behavior as a disposition to act on A^Q whenever the agent is confronted with the question Q that the belief is an answer to.

2. Beliefs as Answers to Questions

2.1 A *question* Q is a partition of logical space Ω, the set of all possible worlds. When two worlds w and v share a cell of this partition, we write $w \sim_Q v$. Any set of Q-cells $A \subseteq Q$ is an answer to Q.

2.2 A *question-directed proposition*, or *quizposition*, denoted A^Q, is an ordered pair $\langle Q, A\rangle$ whose first member is the question Q that A^Q is said to be about, and whose second member is a Q-answer $A \subseteq Q$. The quizposition A^Q is true at a world w if and only if $w \in \cup A$.

2.3 *Example*: Let M be the set of all S-cells that represent a spelling for ‘dreamt’ ending in -MT, and let W be the set of E-cells where ‘dreamt’ is included on the list. Then the quizpositions M^S and W^E have identical truth-conditions, but answer distinct questions.

Again, this can be modeled in two approaches: one that pursues sub-propositional structures and the other that exploits propositional modality.

3. Facing Questions

Now we apply the inquisitive structure to decision-making:

3.1 An *option* is a real-valued function $\mathbf{a} : \Omega \mapsto \mathbb{R}$ from possible worlds to utility values. A decision problem Δ is a finite set of options. A decision problem is an abstract representation of (an agent's) choice.

3.2 The choice Δ *raises* the question Q, or Q *addresses* Δ, just in case for every option $\mathbf{a} \in \Delta$, and every cell $q \in Q$, the outcome $\mathbf{a}(w)$ takes on a constant value for all $w \in q$, denoted '$\mathbf{a}(q)$'. An agent faces the question Q when they make a choice that raises Q.

In other words, a question addresses a choice just in case any complete answer to the question entails what the outcome of each option would be. The utility assignment (relative to options) cannot outstrip the granularity of the partition under Q.

It also means that, all the questions that a choice raises form a downset.

3.3 *Some remarks and points for discussion:*

(i) Hoek here only shows that an agent's questions are directed by her options. In this paper, he has not shown how one's question-structure could affect (say, restrict) the options she has.

(ii) A more adequate representation of the agent's options would not assign utilities relative to possible worlds, but relative to (more corse-grained) possible states. But were we to add question-structure to one's options (which is essentially a matter of practical reason), would it mean that we should do the same to her representation of possible states as well (which is a matter of epistemic reason)? If so, which is more primitive? How are they related?

4. Acting on Answers

Of course, a question-sensitive decision theory is not the center of focus of Hoek's paper. Rather, it suffices for him to derive the following:

4.1 *Classical Belief-Action Principle:* A belief that p manifests in action as a disposition to forego p-dominated options in all decision situations.

Inquisitive Belief-Action Principle: A belief that A^Q manifests in action as a disposition to forego A^Q-dominated options in any decision situation that raises Q.

5. The Practical Problem of Deduction

Now we are in the second part of the paper. Hoek begins by defining the classical picture:

5.1 A *classical information state* is a set of propositions I such that:

(i) *Closure under entailment/necessitation*: If $p \in I$ and q is true at all possible worlds where p is true, then $q \in I$.

(ii) *Closure under conjunction*: If $p, q \in I$, then $(p \vee q) \in I$.

An information state I is *accurate* at a possible world w if and only if all propositions $p \in I$ are true at w; I is *consistent* if and only if it is accurate at some world.

Classical Belief States: An agent X's beliefs form a consistent classical information state B_X and manifest as a general disposition to forego $\bigwedge B_X$-dominated actions.

5.2 Hoek notes that the Classical Belief State is confirmed by the classical belief-action principle, according to which agents behave as if they believed any conjunction of their beliefs, and also that having inconsistent beliefs is impossible for a classical agent. (See his proofs in footnotes 25 and 26.)

Especially, given the following principle:

(Quacks-Like-a-)Duck Principle: If an agent X has the behavioral dispositions that are associated with a belief with a certain content, and moreover X has those dispositions in virtue of their beliefs, then X actually does have a belief with that content

it follows that Classical Belief State is entailed by the classical belief-action principle.

Since it is unhelpful to reject the Duck principle, Hoek suggests that we have to reject the classical belief-action principle (to preserve our judgment that Classical Belief States has to go).

6. Quizpositional Mereology

6.1 One question Q *contains* (or is at least as big as, or entails) another question R if and only if every R-cell is a union of Q-cells. R is *part* of Q if and only if Q contains R. Equivalently, R is part of Q just in case $w \sim_R v$ whenever $w \sim_Q v$.

6.2 The *overlap* (or meet) of two questions Q and R is the biggest question that is both part of Q and part of R. Two questions overlap if and only if their overlap is not equal to the empty question $\{\top\}$.

6.3 The conjunction of a Q-answer A and an R-answer B is the QR-answer $AB = \{(a \cap b) : a \in A \text{ and } b \in B\} \setminus \{\emptyset\}$. The conjunction of the quizpositions A^Q and B^R, written AB^{QR} or $A^Q \wedge B^R$, is the quizposition $\langle QR, AB \rangle$.[1,2]

[1] QR is the partition $\sim_Q \cap \sim_R$.

[2] Quizpositional negation is defined as

$$\neg A^Q := \langle Q, Q \backslash A \rangle$$

while disjunction defined as

$$A^Q \vee B^R := \neg(\neg A^Q \wedge \neg B^R) = \langle QR, A \cup B \rangle$$

6.4 A quizposition AQ *contains* a quizposition BR if and only if Q contains R and AQ entails BR (that is, $\cup A \subseteq \cup B$); alternatively, we can say BR is *part of* AQ. If R is any part of Q, the maximal R-part of AQ, written AQ/R, is the part of AQ about R that contains all other parts of AQ about R.

6.5 An *inquisitive information state* is a set of quizpositions I subject to the following closure conditions:

(i) *Closure under parthood:* if $AQ \in I$ and AQ contains BR, then $BR \in I$.

(ii) *Partial closure under conjunction:* If $AQ, BR \in I$, and Q contains R, then $AB^Q \in I$.[3]

6.6 The *domain* of I, denoted $\mathcal{D}_I$, is the set of all questions about which I contains at least one quizposition. For any $Q \in \mathcal{D}_I$, I's *view* on Q, denoted $I(Q)$, is the strongest quizposition V^Q in I that is about Q.[4]

6.7 *Inquisitive Belief States:* An agent X's beliefs form a coherent inquisitive information state B_X, and manifest themselves in a disposition to forego $B_X(Q)$-dominated options when confronted with a question $Q \in \mathcal{D}_{B_X}$.

[3] A question: I understand that an agent may have different questions *across time*, but what really stops a non-fragmanetalized agent from having full closure under conjunction at *a certain time t*?

[4] It follows from the closure conditions above, then, that if R is part of Q and the agent's view on Q is V^Q, then their view on R must be V^Q/R, the view that rules out all and only those R-possibilities that A^Q rules out.

7-10. Doxastic Daisy Chains, Failure of Deductive Closure, Inconsistent Beliefs, Necessary Truths and Dutch Books

7.1 *Overlapping Views:* If I is an inquisitive information state, and two questions $Q, R \in \mathcal{D}_I$ have a common part S, then $I(Q)/S = I(R)/S = I(S)$.

7.2 It follows that "views on disjoint questions may be linked by one or more daisy chains of intermediate views, where each link in the chain overlaps its neighbors. A change in view at one end can percolate throughout the daisy chain." For example, see Hoek's illustration on p.137:

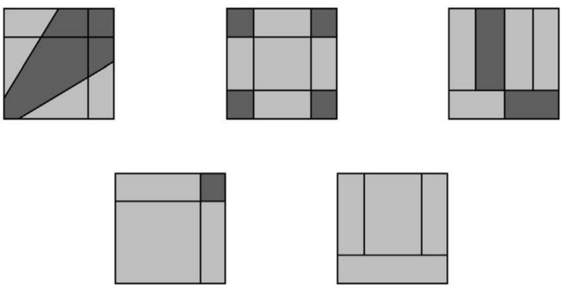


Figure 3. A daisy chain of interlocking views

7.3 *Some Remarks:* Based my question in side note 4—would it be a more adequate account for modeling either states of an agent at different times or states of different agents in a group (*pace* Hoek's defense against the 'fragments'-view in p.139)?

It shows more affinity to a 'fragments'-view when he discusses inconsistent beliefs:

> While Mandy's beliefs are *inconsistent*, they are not *incoherent* [...]: Mandy does not believe outright contradictions. This is possible because her beliefs are not fully closed under conjunction. (141)

7.4 A novel contribution from Hoek is the treatment of deductions: deductive accomplishments can often be understood in terms of acquiring a belief in a necessary truth (in terms of updates with quizpositions, i.e. AB^{AB}).

The *update* of an inquisitive information state I by a quizposition A^Q, written $I + A^Q$, is the smallest inquisitive information state containing $I \cup \{A^Q\}$ as a subset.

Updating an inquisitive belief state with a necessarily true quizposition Q^Q, therefore, can yield new beliefs, including new contingent ones. For example:[5]

[5] See Hoek: "Minimal Rationality and the Web of Questions".

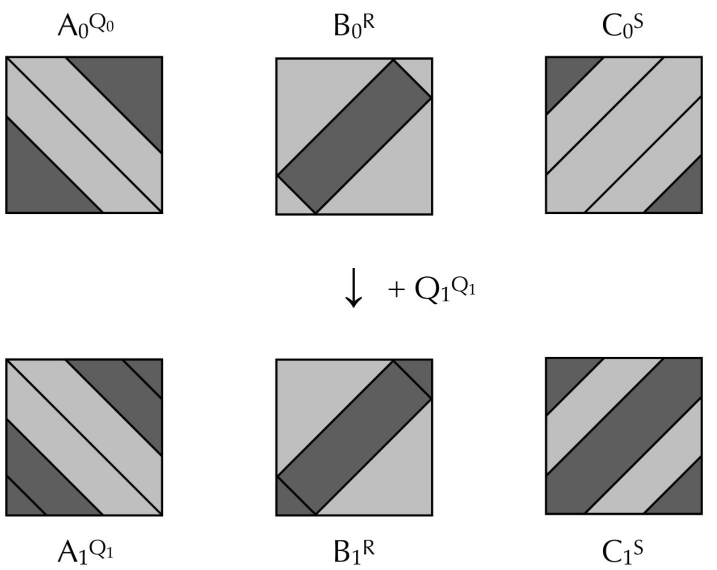


FIGURE 6: A TAUTOLOGICAL UPDATE ON OVERLAPPING VIEWS

Thus, Hoek concludes that deductive inquiry leads to a more cohesive behavior.

7.5 Finally, and in summary, perhaps we can say that the peculiarity of Hoek's 'fragmentish' account of agent's beliefs, apart from his treatment of question-sensitivity, is that it is independently motivated by, and finds its ground in, the belief-action principles.

Say that a *Dutch Q-book* is a Dutch book in which every bet is addressed by the question Q. Hoek's idea is that an agent who believes Q^Q is disposed to avoid Dutch Q-books (and thus remains coherent).

A. Some further points I wish to discuss ...

Can we infer about questions (and question-sensitive) without using quizpositions?

Note that usually, our talk of the agent's questions is couched in metalanguage (e.g. cells, partitions, etc.). Quizpositions offer an alternative that enables us to move back and forth between objective and metalanguage. So, we can just attribute a quizposition to a subject as her doxastic content. (Other similar alternatives include e.g. inquisitive semantics.)

But can we achieve this in a more conservative way?

Note that some easier, and more traditional, ways do not quite deliver what we want: e.g. say that A (partially) answers Q iff $[\sim_Q]A$.[6]

[6] Cf. e.g. van Benthem and Minică (2012), "Toward a Dynamic Logic of Questions."

So, if we want to express that p is part of a's total questions Q_{total}, by using a sentence like $\mathsf{Q}_a p$, where 'Q_a' is intended as a modal operator, then, we say that $\mathsf{Q}_a p$ is true at w iff $\exists s \subseteq Q_{total}$ such that $|p| = \cup s$.[7]

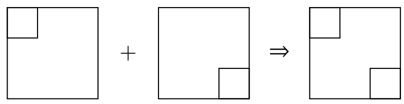
[7] If Q_{total} is closed under union, then $\mathsf{Q}_a\phi$ is true at w iff $|p| \in Q_{total}$. The truth-condition of its dual is defined as follows: $\widehat{\mathsf{Q}}_a\phi$ is true at w iff $W\backslash|\phi| \notin Q_{total}$. Thus, $|\widehat{\mathsf{Q}}_a\phi| = W\backslash|\mathsf{Q}_a(\neg\phi)|$.

This might be closer to what we want. If $|p| = \cup A$, then, the quizposition A^Q is true at a world w just in case $\mathsf{Q}_a p \wedge p$.

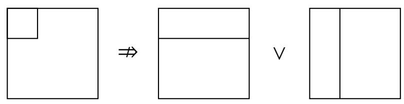
However, there are still some issues to deal with:

(i) Note that the modality expressed by 'Q_a' is very weak and obviously non-normal. While it agglomerates over conjunction, basically it does not generally distribute over any logical connection.[8] But there is no regular relational structure to validate this.

[8] This can be shown as follows (clearly, Hoek would reject even this agglomeration over conjunction.):



But:



(ii) Also notice that the agent's total question Q_{total} as presented above is *not* world-relative. Thus, $\mathsf{Q}_a p$ is if true necessarily true (true throughout the agent's logical space Ω_a).

Of course, we can add world-relativity into the framework such that, at different worlds, the agent's total question partition could be different. (What would be the motivation for this? And what would be the motivation against world-relativity?) But if we do so, this will generate even less modal regularity for Q_a. In fact, the framework will just be an ordinary neighborhood frame.