Cian Dorr, "Quantifier Variance and Collapse Theorems", section 1-4

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1. Quantifier Variance

(QV) There are different candidate meanings for quantifiers.

For (QV) to be philosophically interesting, the candidate meanings must be *quantifier* meanings: they must *behave like quantifiers* in inferences.

QV is often used to motivate metaontological deflationism: apparently different ontological views are "saying the same thing" in different ways. That requires all relevant parties using *quantifiers* in their language.

*This is essentially saying that they follow the same set of inference rules. Rules do not fully determine meaning.

2. Collapse Theorems of $\vee$

Collapse theorems claim that inference rules do determine meaning (up to mutual entailment). Dorr ilustrates this by disjunction.

Theorem 1: We may prove a collapse theorem with respect to an uninterpreted language S: let $\vee_1$ and $\vee_2$ be two syntactic objects that follow the usual inference and elimination rules of $\vee$, then:

(1) $\phi_1 \vee_1 \phi_2 \vdash \phi_1 \vee_2 \phi_2$

(2) $\phi_1 \vee_2 \phi_2 \vdash \phi_1 \vee_1 \phi_2$

Proof for (1). $\phi_1 \vdash \phi_1 \vee_2 \phi_2$, $\phi_2 \vdash \phi_1 \vee_2 \phi_2$, Therefore, by $\vee_{Elim}$ of $\vee_1$, $phi_1 \vee_1 \phi_2 \vdash \phi_1 \vee_2 \phi_2$ Proof for (2) is symmetric to (1).

This shows that there is at most one symbol playing the inferential role of $\vee$ in a syntax. Now let there be two languages L_1 and L_2. By Theorem 1, each language contains just one symbol for $\vee$.

The syntactic proof above won't help block the possibility of semantic variation, because syntactic rules cannot be applied *across* languages.

But this is not yet an argument for the claim that "$\vee$" has the same *meaning* in languages L_1 and L_2, both with the same syntax! The obstacle is that we cannot directly use inference rules on a formula in another language.

When ϕ_1 and ϕ_2 are formulas in L_1, the only relevant inferential rule is $\vee_1$-intro, so we cannot get any meaningful claim about $\vee_2$.

Local vs. Global rule-following: to address the difficulty we need to define rule-following on the level of propositions . The "local" definition (p. 509) captures the inferential behavior or $\vee$ on propositions that are expressible in the relevant languages. The "global" definition (p. 511) defines the inferential pattern on all propositions.

Propositions are the *semantic values* of syntactic structures/sentences, so they transcend specific languages.

Theorem 2: If L_1 and L_2 have the same expressive power (up to mutual entailment), then for any sentences ϕ_1, ψ_1 in L_1 and phi_2, psi_2 in L_2, if ϕ_1 is equivalent to ψ_1 and ϕ_2 is equivalent to ψ_2 then $\phi_1 \vee_1 \psi_1$ is equivalent to $\phi_2 \vee_2 \psi_2$.

The intuition of this "local" version of Collapse is that with the help of expressive equivalence, we can *"translate"* between equivalent sentences in the two languages, and then run the proof for (1) after proper translation.

Theorem 3: If $F(p,q)$ and $G(p,q)$ are both the least upper bound of propositions p and q, then $F(p,q) = G(p,q)$

The intuition for the "global" version: regardless of whether a proposition is expressed (and expressed in whatever form) in a syntax, we may directly define inferential rules on propositions, and those rules determine uniquely the meaning of "disjunction".

We may use Theorem 3 to argue that the meaning of $\vee$ does not vary across languages.

Possible response: Dorr mentions that the only way to block Theorem 3 is to argue that the notion of entailment varies across languages (which means that the "Global" inferential properties we defined are in fact not univocal.)

Here is what I take to be going on. Let "entail" pick out relation R_1 in language L_1 but R_2 in L_2, then when L_1 speakers say "$F(p,q)$ entails r" they mean R_1 while L_2 speakers will mean R_2. Now, we may rigidify "entail" to mean whatever we mean, say R_2, and check whether their quantifiers satisfy the relevant "global" inferential properties defined in terms of R_2. But then whether the L_1 speakers take a sentence to be entailed by some other sentence provides *no evidence* whether that sentence is "really" entailed, in terms of R_2 by the other sentence.

I will skip the "Tarskian" variation for $\vee$ rules (section 3.5 in the paper) because I (and Dorr) consider that as a distraction. If we may define connectives directly on propositions themselves then we don't need to have a heavy weight notion of combined language within which we run the syntactic Collapse argument (like that for (1)).

3. Collapse Theorems of Quantifiers on Closed Sentences

Define the entailment relation on the semantic values of closed sentences, and we may have the "local" (p. 522) vs. "global" (p. 529) versions of the inference rules concerning $\exists$. As before, local inference rules are defined on propositions expressed by sentences, and global inference rules are defined directly on all propositions.

The local version of collapse

Local inference rules are like the regular inference rules in logic textbooks (p. 522)

Theorem 4: If L_1 and L_2 has a name-mapping such that for every sentence ϕ_1 in L_1 there is a sentence ϕ_2 in L_2 equivalent to ϕ_1, and the names in ϕ_2 are the images of names in ϕ_1 via the mapping and vice versa, then if ϕ_1 in L_1 is equivalent with ϕ_2 in L_2, their existential closure are equivalent too.

The intuition for name mapping: we can translate between L_1 and L_2, and the translation preserves names.

Proof. Suppose ϕ_1 is equivalent to ϕ_2, by the local introduction rule of $\exists_1$, $\phi_1 \models \exists_1 x_1 \phi_1^*$. by equivalence between L_1 and L_2 there is a sentence ψ in L_2 that is equivalent to $\exists_1 x_1 \phi_1^*$, and ψ does not contain the image of the relevant constant that was in the place of x_1 in ϕ_1. Now, because ϕ_2 and ϕ_1 are equivalent, $\phi_2 \models \psi$, and ψ does not contain the relevant "image" constant. Then by elimination rule of $\exists_2$, $\exists_2 x_2 \phi_2^* \models \psi$, and ψ is equivalent to $\exists_1 x_1 \phi_1^*$ The converse is parallel.

Two problems about Theorem 4: First, it relies on name mapping but quantifier variantists may claim that different languages have different stocks of names (those who apparently quantify over more things will also have more names in their language). Second, it is not clear that our language satisfies the local introduction and elimination rules in full generality.

PROBLEMS WITH LOCAL INTRODUCTION OF $\exists$ (A) empty names (if a is empty then $\phi(a)$ does not entail $\exists x \phi(x)$). (B)) contingentism ($\forall x(x \neq a)$ is contingently false, but $\exists y \forall x(x \neq y)$ is necessarily false. So $\exists$-elimination makes a contingently false proposition entail a necessarily false proposition, but this cannot hold if entailment is metaphysical necessitation).

THE BEST SHOT FOR THEOREM 4: we apply Theorem 4 not to $\exists$ but to $\exists_\diamond$ where $\exists_\diamond$ is the possibilist quantifier, this will block problem B. Moreover , for the quantifier variantists, difference in the meaning of $\exists$ will typically result in difference in the meaning of $\exists_\diamond$, but Theorem 4 will preclude any meaning variation on $\exists_\diamond$.

The intuition: the possibilist existential quantifier quantifies over all possibilia including those that are not actual.

The example of being huge on p. 526: let "something is huge" be false under one quantifier meaning but true under another quantifier meaning, then "possibly something is huge" could be false under one quantifier meaning but true under another meaning

PROBLEMS WITH LOCAL ELIMINATION OF $\exists$ If entailment is metaphysical necessitation, then "Hesphorus is a gas giant" enatils "Phosphorus is a gas giant" but $\exists x G(x)$ does not. .

Note that in "Phosphorus is a gas giant" there is no occurrence of "Hesphorus", so we may run $\exists$-Elim.

DILEMMA: To save $\exists$-Elim we may consider a super fine-grained notion of propositions and a Tarskian notion of entailment under which "Hesphorus is a gas giant" does not entail "Phosphorus is a gas giant" (because there are Tarskian permutations that changes the reference of "Phosphorus" but not "Hesphorus"). This move blocks the counterexample. But simultaneously it makes Theorem 4 *useless* because in this super-fine-grained conception the idea of expressive equivalence is super demanding, and it is unlikely that quantifier variantists will endorse it.

The global version of collapse

In the global version of collapse we must define the inferential rules of quantifiers directly on the level of propositions/semantic values. The natural thought is that quantifiers are functions from concepts (the semantic values of predicates) to propositions.

To avoid puzzles about identity, we need to define a notion of aboutness under which qualitative concepts are not about any particular object

GLOBAL $\exists$-ELIM: If proposition p predicates concept c of some object and c is not about that object, $p \models F(c)$

GLOBAL $\exists$-ELIM: If proposition p predicates a concept c of some object and entails some proposition that is not about that object then $F(c) \models q$

The intuitive thought: which object it is won't matter.

Theorem 5: If F and G both follow the Global inference rules, and c is a concept not about every object, then $F(c)$ and $G(c)$ are equivalent.

Proof. Let x be some object that c is not about, and let q be the proposition that attributes c to x. By Introduction, q entails G(c), and it is not about x. q is the result of removing F from F(c), so by elimination rule F(c) entails G(c). The converse is parallel.

Objection 1 (Sider): Theorem 5 assumes that there is a common stock of concepts across languages, but that is implausible. Apparently different ontological views will result in different stocks of concepts and objects across languages.

Response (Dorr): This is radical, and also unmotivated, because t undercuts the motivation for QV–why not attribute meaning variations to predicates?

The obvious response from Sider or a quantifier variantist is that differences in predicate meaning are *explained* by differences in quantifier meaning

Objection 2 (Dorr): Even if we grant that all languages have a common stock of concepts, the quantifier variantist can still deny that the quantifiers all have the global inferential properties. Note that the properties are themselves defined by *quantification* over objects, but it is question-begging to assume that all languages quantify over objects in the same way. If each language has its own quantifier meaning, then in each language the clauses for ∃-Elim and $\exists_{I}ntro$ will pick out different properties too. This is just like Dorr's putative response to the collapse argument of ∨.

A simplistic way to think about this: Theorem 5 not only assumes a common stock of concepts but also a common stock of objects. But if QV is true then the notion of objecthood must be revised too.

Dorr's response carries over to "Tarskian" variations of the Collapse argument if the notion of a legitimate variant involves quantification over objects. And it is not clear how to define Tarskian variants otherwise.

Collapse Theorems of ∃ in Open Sentences

Tentative thought: open sentences express propositions relative to variable assignments, so we may define inferential rules relative to variable assignments.

Problem: Analogous to the Objection 2 to Theorem 5, there is no guarantee that different quantified languages will quantify over variable assignments using the same quantifier meaning (which tacitly involves quantification over objects in the domain), so the notion we use to define the relevant inferential properties may fail to be univocal.

Suggestion of Dorr: take open sentences to be something like predicates, and directly define entailment relations on concepts.

The appeal is that we can avoid any explicit or implicit quantification over objects or variable assignments.