This paper is composed of two parts:

1. Criticism that contemporary debate about Composition as Identity (CAI) is misguided.

2. Reconceptualization of parthood relation as inherently potential.

- ***What is CAI?***

  Some definitions: $P$ is a partial order interpreting parthood relation.

  $O(x, y) =_{def} \exists z(P(z, x) \wedge P(z, y))$

  [$x$ and $y$ *overlap* if they have a common part]

  $F(xx, y) =_{def} \forall z(z \prec xx \rightarrow P(z, y)) \wedge \forall w(P(w, y) \rightarrow \exists z(z \prec xx \wedge O(z, w))))$

  [$y$ is a *fusion* of the $xx$ if and only if any one of the $xx$ is part of $y$ and every part of $y$ overlaps at least one of the $xx$]

  – The interesting CAI: Composition just *is* the identity relation. i.e., let $xxCy =_{def} F(xx, y)$ be composition relation, CAI says the following:

  $$\forall y \forall xx(xxCy \rightarrow xx = y) \tag{CAI}$$

  – Baxter's CAI: The stronger and interesting CAI; no mereological assumptions.

  – Lewis's CAI: Composition is *like* identity; needed only for a defense of unrestricted composition.

  – To take CAI at face value is to follow Baxter's intuition. Failure to do so leads to two fallacies.

    1. The building fallacy: composition has a privileged direction from parts to whole.
    2. The inventory fallacy: parts and whole are two ways of referring to the same object.

  – CAI also faces a dilemma even taking it at face value, and Botti's intuition that parts reveals more *metaphysical information* (a notion yet to be clarified) than the whole they compose may rescue it from the dilemma.

- ***CAI's dilemma.***

– Failure: Note first that Collapse is problematic:

$$\forall xx \forall y(F(y, xx) \rightarrow \forall z(z \prec xx \leftrightarrow P(z, y))) \tag{Collapse}$$

If Collapse were true, then something that is not a cat, e.g. a cat molecule, would be one of the cats. However, CAI (along with Plural covering) entails Collapse.[1]

$$\forall x \forall y(P(y, x) \rightarrow \exists zz(F(x, zz) \wedge y \prec zz)) \tag{Plural covering}$$

Responding to the failure: Collapse depends two assumptions that can be given up or restricted:

1. There is the fusion of a given plurality of objects.

   If there is not the fusion of a given plurality of objects, then trivially every part of the fusion is in the plurality. Hence making Collapse harmless.

2. There is a plurality of any given objects.

   This is guaranteed by plural logic's comprehension principle:

   $$\exists xx \forall y(y \prec xx \leftrightarrow \varphi(y)) \tag{Plural Comprehension}$$

   Restricting $\varphi$ might avoid the failure.

Problem for the response: modifying either assumption leads to the boredom part of the dilemma. Worse, the boredom is independent of Collapse.

– Boredom: CAI and Plural covering imply mereological nihilism.[2]

$$A(x) =_{def} \neg \exists y(y \neq x \wedge P(y, x)) \tag{Atom}$$

$$\forall x A(x) \tag{Nihilism}$$

Botti moves on to provide an framework that solves the dilemma, by going modal.

- ***Reconceptualizing parthood—parts as* interpretational possibilities *for wholes*.**

[1] *Proof* for Plural covering. Assume $P(y, x)$ and let $zz$ be those things that are either $x$ or $y$. Then $y \prec zz$, and $F(x, zz)$, for each of the $zz$ is part of $x$—for $\forall z(z \prec zz \rightarrow (z = x \vee z = y))$ holds—and each part of $x$ overlaps one of $zz$—that is, $x$ itself. ■

*Proof* for Collapse (reiterating Botti (2021, p. 4553)). Assume the antecedent. The left-to-right direction of the bi-conditional follows trivially by the definition of fusion. The right-to-left direction: assume $P(z, y)$. By Plural covering, there are some $yy$ such that they compose $z$ and $y$ is one of them. By CAI and symmetry and transitivity of identity, however, $yy$ is identical to $xx$. By Indiscernibility of Identicals, then, $y$ is one of the $xx$, namely $y \prec xx$. ■

[2] *Proof*. Assume $P(x, y)$. By Plural covering, there are $zz$ such that (i) $x \prec zz$ and (ii) $F(y, zz)$. By CAI, (iii) $y = zz$. Replacing $zz$ in (i) with $y$, we get (iv) $x \prec y$, "$x$ is one of the $y$". Since both $x$ and $y$ are singular variables, the most natural reading is that $x$ is identical to $y$. Hence nothing has something distinct from itself as its part, and it follows that everything is an atom. ■

- An important distinction: pluralities are extensionally defin*able* as opposed to defined.
- Plural Comprehension raises a similar problem in the vicinity of set theory:
  1. $FORM(xx, y) =_{df} \forall u(u \prec xx \leftrightarrow u \in y)$
  2. Collapse* : $\forall xx \exists y FORM(xx, y)$
  3. Plural Comprehension: $\exists xx \forall y(y \prec xx \leftrightarrow \varphi(y))$
  4. Paradox from (2) and (3): $\forall u(u \prec tt \leftrightarrow u \notin u)$, which implies $t \in t \leftrightarrow t \notin t$.

  One solution to the paradox: Treating set-theoretic claims as inherently modalized.
- Linnebo's set-theoretic potentialism: the domain of quantification for sets *can* be expanded.

  In particular, the quantifiers $\forall$ and $\exists$ in set-theoretic claims should be replaced by $\Box\forall$ and $\Diamond\exists$.

  The potentialized Collapse*, $\Box\forall xx \Diamond\exists y FORM(xx, y)$, says that for any plurality of objects $xx$, there can be a set formed of exactly those objects.

  When $\varphi$ in Plural Comprehension stands for, e.g., being self-identical, the resulting plurality is *extensionally non-rigid* or extensible, for its extension is enriched whenever a new set is formed.

  This would resolve the paradox if we let $\varphi$ exclude those properties that result in extensionally non-rigid pluralities.
- Botti's mereological potentialism: composite objects as extensionally non-rigid pluralities.

  **The Model**:

  * Non-extended quantifiers $\forall$, $\exists$, and their extended counterparts: $\forall^+$, $\exists^+$.
  * Extensible domains of quantification $D_1$,. . . , $D_n$ ordered by a partial order $\leq$.
  * Modal operators $\Box$ and $\Diamond$ defined as follows:
    · $\Diamond A =_{df} \exists^+ D_i \leq n$ such that $A$ is true at $D_i$.
    · $\Box A =_{df} \forall^+ D_i \leq n$ such that $A$ is true at $D_i$.
  * $\varphi(a)$ is a stable formula if: $\varphi(a)$ iff $\Box\varphi(a)$, and $\neg\varphi(a)$ iff $\Box\neg\varphi(a)$

  **The Framework**:

  * Parts exist within the expanded quantification domain.

    * Whenever we say that some objects are parts of an object, we expand the interpretation of our quantifier so as to capture the existence of the parts of the object we have started from.
    * The only $\varphi(a)$ that's stable is *being a certain portion of reality*.
    * The extensible domains ordered by $\leq$, $\langle D, \leq \rangle$, is an *information ordering*, and the order can be interpreted as the relation of *being more informative than*.

  Thus conceived, the world itself is the bottom of the ordering, and there is no top; and the only stable $\varphi$ is the property *being more informative than the world*.

  A possible example for the first three elements in the ordering:

    * $D_1$ consists of the world itself, considered as an extended simple.
    * $D_2$ consists of some parts of the world and the world itself, introduced via lifting.
    * $D_3$ consists of some parts of the parts in $D_2$ and the world introduced via lifting.

  – Solving CAI's problem.

  Consider the fusion of cats in $D_n$, and its cat-parts in $D_{n+1}$. There is a plurality $xx$ of the cats in $D_{n+1}$ but no cat molecule is in the same domain of quantification so we cannot apply Plural Comprehension to get a plurality of cat molecules. So Collapse is no longer problematic.

- ***From CAI to Composition as Analysis.***

Metaphysical information is what we can say about the world, and, in particular, what the world is composed of. For some things to be more metaphysically informative than the world itself taken as an extended simple requires that the things be parts of the world. Composition is identity in the sense that fusion and parts of the fusion take up the same portion of reality, although the latter is more informative than the former.

***Quistions***: It seems that the dilemma is resolved in a problematic way, because each domain is not even a mereological model. If we close each domain under fusion, Collapse remains a problem.

How do other mereological theses fit into this potentialist conception of mereology? In particular, given the stipulative nature of interpretational modality, is Gunk, the thesis that everything has proper part, implied? Is there a way for atomism and gunk be compatible in this model?