If p, then p!

by Matthew Mandelkern, forthcoming in *The Journal of Philosophy*

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Notations & Abbreviations

- atoms: $A, B, C...$
- sentences: $p, q, r...$
- connectives: $\wedge, \vee, \neg$ (all classical)
- material conditional & biconditional: $\supset$ & $\equiv$
- conditional (if...then...): $>$ (especially $>_i$ for indicatives, and $>_s$ for subjunctives)
- semantic entailments: $\Gamma \models p$
  iff p is true in every world in every model where all the element of Γ are true.

- ***Identity***: $\models p > p$
- ***Import-Export (IE)***: $\models (p > (q > r)) \equiv ((p \wedge q) > r)$
- ***Very Weak Monotonicity (Mon)***: if $\models p > p$ and $p \models q$, then $\models p > q$

> *Monotonicity*: if $\models p > q$ and $q \models r$, then $\models p > r$

- *Identity* + *Mon* = ***Logical Implication principle (LI)***: if $p \models q$ (i.e. $\models p \supset q$), then $\models p > q$

> *LI* is strictly weaker than" $p \supset q \models p > q$ ",
> but *Multi-Premise LI* (if $\Gamma, p \models q$, then $\Gamma \models p > q$) is strictly stronger.

- ***Ad Falsum***: $\{p > q, p > \neg q\} \models \neg p$

> Corollary: if $\models p > q$ and $\models p > \neg q$, then $\models \neg p$

Part 0 - Introduction & backgrounds

- McGee 1985:
  - [Counterexamples to *MP*] & [validate *IE* + *MP* + *LI* = material conditional] >> [reject *MP*, keep *IE*]

> 1. If a Republican wins the election, then if it's not Reagan who wins, it will be Anderson.
> 2. A Republican will win the election.

> 3. If it's not Reagan who wins, it will be Anderson. (1,2 MP)
>
>   NO! It will be Carter.
>
> ---
>
> 1. If that creature is a fish, then if it has lungs, it's a lungfish.
> 2. That creature is a fish.
> 3. If it has lungs, it's a lungfish. (1,2 MP)
>
>   NO! it's a porpoise.

- a semantics that reject *Identity*

  > sentences are evaluated relative to 2 parameters:
  > the Stalnakerian selection function f,
  > and a set of sentences Γ, which gathered successive conditional antecedents.
  >
  > $w \in [p > q]^{\Gamma}$ iff $w \in [q]^{\Gamma \cup \{p\}}$
  >
  > $w \in [A]^{\Gamma}$ iff $f(\bigcap_{p \in \Gamma} [p]^{\varnothing}, w) \in [A]$
  >
  > ...

- Mandelkern 2020:
  - *IE* + classical conjunction + *Nothing added* + *LI* + *Equivalence* + *Quodlibet* = *Ex Falso*

    > *Nothing added*: if $\models p \supset q$, then $\models (p > (q > r)) \equiv (p > r)$.
    >
    > *Quodlibet*: as long as p is conditional-free, p ^ ~q is nowhere true, and thus entails everything.
    >
    > *Equivalence*: if $\models (p > r) \equiv (q > r)$ no matter what r is, then $\models p \equiv q$.
    >
    > *Ex Falso*: $\neg(p > q) \models p$. (disastrous result of material conditional)

  - *Restricted IE*: if p contains NO conditional, then $\models (p > (q > r)) \equiv ((p \wedge q) > r)$

Part 1 - Why should we reject IE?

- why several theories of conditional invalidate *Identity*?
  Because they validate *IE*.
- **validate *IE* + *Identity* + *Mon* + *Ad Falsum* = material conditional**

  > proof...

- The natural language condition "If...then..." is NOT the material conditional.

  > $\neg(p \supset q)$ is equivalent to $p \wedge \neg q$, but:
  >
  > 1. It's not the case that, if Patch is a rabbit, she is a rodent.
  >   It's not the case that, if Patch had been a rabbit, she would had been a rodent.
  >
  >   **Both TRUE** regardless of whether Patch is a rabbit. (as rabbit is not rodent)
  > 2. Patch is a rabbit and not a rodent.
  >
  >   **FALSE** if Patch is not a rabbit.

- which one should we reject? *Mon*, *Ad Falsum*, *Identity*, or *IE*?
  - *Mon* or *Ad Falsum*? No.

> Comparing McGee(1985): *Ad Falsum* is strictly weaker than *MP*.

- *Identity*? No.

  > Arguments:
  >
  > 1. Many reject $\models \bot > p$, but even *identity*$_\bot$(whenever p is consistent, $\models p > p$) can lead to a similar result.
  > 2. No intuitive counterexample to *Identity*, even in *IE*-validating theories.
  >   But there are intuitive counterexamples to *IE*.

- **Thus, we should reject *IE*.**

- **There are counterexamples to *IE* for subjunctives, but unfortunately none for indicatives.**

  > a die which is either weighted towards evens or odds; we don't know which.
  >
  > For subjunctives:
  >
  > 1. If the die had been thrown and landed four, then if it hadn't landed four it would have landed two or six. (True)
  > 2. If the die had been thrown and landed four and it hadn't landed four, it would have landed two or six. (False, or incoherent)
  >
  > But for indicatives:
  >
  > 1. If the die was thrown and landed four, then if it didn't land four it landed two or six. (False)
  > 2. If the die was thrown and landed four and it didn't land four, it landed two or six. (False)
  >
  > ---
  >
  > 1. If the exams had been marked, then if the faculty had gone on strike, then the exams would still have been marked. (can be False)
  > 2. If the exams had been marked and the faculty had gone on strike, then the exams would still have been marked. (obviously True)
  >
  > But for indicatives:
  >
  > 1. If the exams were marked, then if the faculty went on strike, then the exams were still marked. (obviously True)
  > 2. If the exams were marked and the faculty went on strike, then the exams were still marked. (obviously True)

Part 2 - Why there's no counterexample?

- why we can't find intuitive counterexamples to IE for indicatives?
  Because though IE is not valid, it is somehow presupposed for indicatives.

- Strawson concepts

  > Context is a set of worlds.
  > For simplicity, assume we often talk about context worlds as epistemically accessible worlds.

  - Strawson entailments

        - Γ *Strawson entails* p (write $\Gamma \models_S p$)
          iff for any context c, world $w \in c$, if the presuppositions of all the members of Γ and of p are satisfied in $\langle c, w \rangle$, then if all the members of Γ are true at $\langle c, w \rangle$, so is p.
    - Strawson valid
        - p is *Strawson valid* iff $\models_S p$

          > when $p \equiv q$ is Strawson valid, call p and q are *Strawson equivalent*.
          >
          > notice: $\models_S p \supset q$ is NOT equivalent to $p \models_S q$

        - Example: "She is female" is not logical valid, but is Strawson valid.
    - Strawson informationally valid
        - $p \supset q$ is *Strawson informationally valid*
          iff for any context c, if the presuppositions of p and of q are satisfied in $\{\langle c, w \rangle | w \in c\}$, then if p is true through out c, p is true through out c.

          > when $p \supset q$ and $q \supset p$ is Strawson informationally valid, call p and q are *Strawson informationally equivalent*.
          >
          > $p \supset q$ is Strawson informationally valid == p *Strawson informationally entails* q ?

    - **We will show that: though IE is not logical valid, it is Strawson (informationally) valid for indicatives, but not for subjunctives.**
- presuppositions for indicatives
    - this idea will be develop by an account of the differences between indicatives and subjunctives
    - Stalnaker's semantics: only differ on the selection functions

      > 1. f treats contextually possible worlds as being closer to each other than any other worlds
      >   i.e. for all world $w \in c$, if there's a p-world in c, then $f_i(p, w) \in c$
      >   - which leads to "or-to-if": when leave open $\neg p$ and accept $p \vee q$, there's $\neg p >_i q$
      > 2. the antecedent p is compatible with c

    - the indicative constraint
      $p >_i q$ presupposes: at a context c, for all $w \in c$, $f_i(p, w) \in c$
- local context & locality constraint
    - the die case

      > a die which is either weighted towards evens or odds; we don't know which.
      >
      > - global context:
      >   we don't know which, and we don't know whether the die was thrown.
      >
      > If the die was thrown and landed four, then if it didn't land four it landed two or six.
      >
      > - global context:
      >   we don't know which, and we don't know whether the die was thrown.
      > - local context for the clause "if it didn't land four it landed two or six":
      >   we know the die was thrown and landed four, which means it weighted

> towards evens.

- Calculating local contexts

  > - the local context for a conditional's consequence entails the antecedent
  > - the local context for the right conjunct entails the left conjunct
  > - the local context for the scope of a quantifier entails its restrictor

- **locality constraint**:
  $p >_i q$ presupposes: at the conditional's local context κ, for all $w \in \kappa$, $f_i(p, w) \in \kappa$

  > Example of the violation of the locality constraint:
  >
  > For embedded conditionals $\neg p \wedge (p >_i q)$
  > let c^p be the set of worlds in c where p is true and has its presuppositions satisfied.
  > $\kappa = c^{\neg p}$, $f_i(p, w) \in \kappa$ is contradict with *Success*

- a bounded theory, instead of dynamic ways: avoid invalidating *Identity*?

- **the locality constraint entails that *IE* is Strawson (informationally) valid for indicatives**
  - Strawson informationally valid: i.e. for all context s, when locality constraints are all satisfied, $p >_i (q >_i r)$ is true throughout s iff $(p \wedge q) >_i r$ is true throughout s.

    > proof...

  - Strawson valid: i.e. $\models_S (p >_i (q >_i r)) \equiv ((p \wedge q) >_i r)$

    > proof...

- Conclusion:
  If $p >_i (q >_i r)$ and $(p \wedge q) >_i r$ are both felicitously used, **its impossible to accept one without the other. Thus we can not give out intuitive counterexamples.**