Bacon on Logical Combinatorialism

Structura

Minzhe Li

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1 Introduction

- The book of the world

  ... the fundamental properties and relations are the primitive constants in the language of reality, from which all other properties and relations can be defined; they are the vocabulary God would need in order to write the "book of the world" (p. 539)

- Structure

  - Logical necessity: section 1-3
  - Structure built on the fundamental: section 4
  - A model of reality: section 5

- The Broadest Necessity (2018a p. 741)

  - Weak necessity: $\lambda X X\top$ (Jess said that)
  - Necessity: ($Nec :=$) $\lambda Y \forall X(X\top \to XY\top)$
  - Broadness: $\lambda Y \lambda Z \forall X \forall y(Nec(X) \to X(Yy \to Zy))$

- Identity (fn. 18; 2018a, sec. 3)

  - Leibniz Equivalence: $=_\sigma := \forall_{\sigma\to t} X(XA \to XB)$
    * Identity: $A = A$
    * Substitution (Leibniz's law): $A = B \to (\phi \to \phi[A/B])$
  - Rule of Equivalence: If $\vdash A \leftrightarrow B$ then $A = B$ (Stronger than Booleanism: $(A = B) = (B = A)$; for type t)
  - Functionality: $\forall x(Xx = Yx) \to X = Y$

2 Logical Necessity

- A necessity that stands to reality as logical truth stands to language.
- Logical truth (consistency)
  - Bolzano: A sentence is a logical truth iff all of its substitution instances are true. (language-dependent)
  - Bolzano-Tarski: A sentence $A(c_1...c_n)$ is a logical truth iff $\forall x_1...x_n A(x_1..x_n)$
- Logical necessity as the broadest necessity
  - $\Box := \lambda p(p = \top)$
  - S4
- Quantified Logical Necessity as further constraint
  - **Quantified Logical Necessity**: $\forall X \forall \bar{z} Pure(X) \wedge Fun(\bar{z}) \rightarrow (\Box X(\bar{z}) \leftrightarrow \forall \bar{x} X(\bar{x}))$.
    The dual version $\forall X \forall \bar{z} Pure(X) \wedge Fun(\bar{z}) \rightarrow (\Diamond X(\bar{z}) \leftrightarrow \exists \bar{x} X(\bar{x}))$
  - Fundamental: metaphysically simple; denoted by nonlogical constants (pair-wise distinct, cf. fn. 12 & 13);
  - Pure: purely logical; denoted by expressions with only logical vocabulary; section 5 purity.
  - Intuitively, it means: "if there are some things that occupy a given logical role, then it's possible that the fundamental things have that role, and, conversely..." (p. 547)
  - Objection: how to pin down the three notions?
    Logical necessity can be defined as $\lambda p \forall X(Xp \leftrightarrow X\top)$; purity is intuitive (or see section 5, p. 578); fundamentality is thus linked to the two via quantified logical necessity.
- Combinatorial ideas incorporated
  - No Brute Necessities: $\Diamond A$ (where A is logically consistent sentence).
  - Pattern: Any actually instantiated pattern is possibly instantiated by the fundamental relations.
  - Contentious for metaphysical necessity; extremely plausible for logical necessity.
  - Case: love triangle; inaccessibility.
- An anomaly: the contingency of distinctiveness
  - the necessity of identity: Leibniz's law.
    Proof: $\Box x = x$, and by Leibniz'z law we have that $x = y \rightarrow (\Box x = x \rightarrow \Box x = y)$, and hence $x = y \rightarrow \Box x = y$.
  - $Fun(x) \wedge Fun(y) \rightarrow \Diamond x = y$

- No (generalizations of) symmetry: (weak Brouwerian axiom)
  Proof: From the necessity of identity we have that $\neg\Box x = y \rightarrow \neg x = y$, and by necessitation and K, $\Box\neg\Box x = y \rightarrow \Box\neg x = y$, and the contraposition of B says $\neg x = y \rightarrow \Box\neg x = y$, therefore we have that $\neg x = y \rightarrow \Box\neg x = y$. Proof from B^n is similar. For $B^{<\omega}$, the necessitation of the necessity of identity, combined with $B^{<\omega}$, give rise to $\Diamond x = y \rightarrow x = y$, the contraposition of which is the desired result. The correspondence between B ($B^n/B^{<\omega}$) and the frame property of (the generalization of) symmetry is easy to verify. (?: transitive)
- No convergency. ($G1$ is invalid ($\Diamond\Box A \rightarrow \Box\Diamond A$))
- Response
  * Metaphysical necessity may well be S5.
  * Logical necessity is introduced via a particular logical role, not by any pretheoretical intuition.
  * No surprise given the analogy between reality and language (no distinctness statement involving different names is a logical truth) $((\Box\Diamond\exists xFx) \vee (\Box\Diamond\neg\exists xFx))$

- Advantages: It is simple to state, strong, and parsimonious (does not rely on set theory).

3 Structure built on the fundamental

- Unstructured entities vs. structured entities
- Two characteristic structural ideas
  - Any proposition, property etc. can be decomposed uniquely into fundamental constituents via logical operations.
  - The fundamental are simple and cannot be defined nontrivially out of other fundamental constituents.

3.1 Decomposition

- Metaphysical definability: $MD(y, \bar{x} := Pure(X) \wedge y = X\bar{x})$
  Metaphysical Definability stands to reality as the notion of definability stands to language.
  - Not confined to the fundamental.
- The uniqueness of decomposition:
  Quantified Separated Structure : $\forall XY\forall\bar{z}(Pure(X) \wedge Pure(Y) \wedge Fun(\bar{z}) \rightarrow (X\bar{z} = Y\bar{z} \rightarrow X = Y))$.
  - It follows from Quantified Logical Necessity and (Modalized) Functionality.

- (*) $F(a_1...a_n) = Ga_1...a_n \to F = G$ provided F and G are closed and contain only logical vocabulary and $a_1...a_n$ are distinct fundamental constants.
- **Separated Structure** : $Fc = Gc \to F = G$ provided c is a fundamental constant that doesn't appear in F or G. (?: $\lambda F\bar{a}c = \lambda G\bar{a}c$)
- Separated Structure as a restricted case of Structure: Mary loves Mary.
- Restricted uniqueness (quasi-uniqueness): converse, forgetfulness and duplicates: therefore $\bar{z}$ contains no duplicates, and, as a sequence, is given in particular order (no converse) and particular length (no forgetfulness).

- The existence of decomposition: given a finite list of types, $\sigma_1...\sigma_n$
  Fundamental Completeness : $\forall x \exists Y \exists \bar{z}(Pure(Y) \wedge Fun(\bar{z}) \wedge x = Y\bar{z})$
  - Concerns: inconsistency; infinite list of types; vagueness (or one may allow some of the fundamental be vague)

3.2 Simple element

- **Fundamental Independence:** $\forall \bar{x}y \forall X(Fun(\bar{x}y) \wedge Pure(X) \to \neg X\bar{x} = y)$ (fn. 53)
- $Fun(R) \wedge Fun(S) \wedge \neg R = S \to \neg(R = \lambda xySyx)$. $(PRS = R; QRS = CS)$
  Symmetry between less massive than and more massive than? Fundamental basis.
- $Fun(R) \to \neg(R = \lambda xyRyx)$. $(IR = CR)$
  Conjunction? Pure but not fundamental.
- Purity of pure and fundamentality of fundamental (Melianism)
- Fundamental and Pure can not be both pure: $D := \lambda p \forall Xr(Pure^{t \to t}(X) \wedge Fun^t(r) \wedge p = Xr \to \neg Xp)$, consider $D(Dr)$ (where r is a fundamental proposition), it is true iff the result of substituting Dr for r in Dr is false.

4 A model of reality

- A metaphysical substitution maps each fundamental element to an arbitrary element to an arbitrary element of the same type and nonfundamental element to the result of replacing the fundamental constituents in it according with the former step.
- Some minimal laws: 1; $i \circ j$; $i(fa) = (if)(ia)$.
- Logical possibility: substitutions stand to logical possibility as possible worlds stand to metaphysical possibility – we may simply identify propositions with sets of substitutions.
  - The identification ensures Booleanism.

- ip is true iff p is true at i
- p is true at 1 iff $1 \in p$
- What is ip?: $j \in ip$ iff $1 \in j(ip)$ iff $1 \in (j \circ i)p$ iff $j \circ i \in p$

- Fundamentality: Constants freely generate the language: any function that takes each constant of the language to a closed expression of the same type can be extended to a unique substitution of the language.
  - No complex expression: there may well be a function which maps $p \wedge q$ to $\neg r$ (ensured by existence of decomposition)
  - The list of constants must be complete: otherwise there will be multiple substitutions. (ensured by uniqueness of decomposition)
- Purity: An element $a \in D^\sigma$ is pure if and only if $ia = a$ for every substitution i.
- Logical Necessity: for pure f and fundamental $a_1 ... a_n$, we have that $fa_1 ... a_n$ is logically necessary iff for every substitution i, $i(fa_1 ... a_n)$ is true iff for every substitution i, $f(ia_1)...(ia_n)$ is true iff for every n-tuple $k_1 ... k_n$ of elements of the domain, $fk_1 ... k_n$ is true.