Goodman, Grounding, and Generalizations

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1. *Krämer's puzzle*

Let's start with a second-order propositional language $\mathcal{L}$:

$$p \mid \neg\phi \mid \phi \wedge \psi \mid \phi \vee \psi \mid \phi \rightarrow \psi \mid \phi \leftrightarrow \psi \mid \forall p\phi \mid \exists q\psi \mid \phi \prec \psi$$

We use $\prec$ for *partial* ground.[1] $\mathcal{L}$ is equipped with some axioms of classical quantification theory and the inferential rule *modus ponens*:

[1] According to Kit Fine's terminologies ([REF]), this notion of ground is also factive, mediate and strict.

PC All theorems of propositional calculus;

UI $\forall p\phi \rightarrow \phi[\psi/p]$, where ψ is free for p in ϕ;

Dual $\forall p\neg\phi \leftrightarrow \neg\exists p\phi$;

mp Infer ψ from $\phi \rightarrow \psi$ and ϕ.

A conventional wisdom: true generalizations are partially grounded in their true instances. This conventional wisdom can be regimented in $\mathcal{L}$ as follows:

(1) $\forall p(\phi \rightarrow \phi \prec \exists p\phi)$;

(2) $\forall p\phi \rightarrow \forall p(\phi \prec \forall p\phi)$.

But given our logic, (1) is unfortunately inconsistent with another tenet for most grounding theorists: no proposition grounds itself![2]

[2] Proof: By Dual, (3) amounts to $\forall p\neg(p \prec p)$, which by UI implies $\neg(\exists p\, p \prec \exists p\, p)$. Applying UI to (1) also gives us $\exists p\, p \rightarrow \exists p\, p \prec \exists p\, p$: $\exists p\, p$ is just $p[\exists p\, p/p]$. Note that the truth of UI and Dual guarantees $\exists p\, p$, and therefore $\exists p\, p \prec \exists p\, p$. A contradiction.

(3) $\neg\exists p(p \prec p)$.

This is Krämer's puzzle ([REF]), a higher-order variant of a notorious puzzle firstly introduced in Fine [REF]. To solve this puzzle, it seems we have to reject at least one principle mentioned above, either a logical one or a ground-theoretical one.[3] But which one?

[3] A similar logical conflict between (2) and (3) will arise against some additional principle of grounding.

2. *Fritz's puzzle*

Let's consider a richer language $\mathcal{L}^+$: it also includes variables $X, Y, Z, \ldots$ of sentential operators and the corresponding quantifiers $\forall X, \exists X, \forall Y, \exists Y, \forall Z, \exists Z, \ldots$, plus two additional binary connectives $\prec_i$ and $=$ for *immediate* partial ground and propositional identity respectively. A new puzzle, i.e., Fritz's puzzle, that has nothing to do with the irreflxivity of grounding, can be raised in this language with a stronger logic, containing three more axioms and closed under one more rule:

Ref $\phi = \phi$;

LL $\phi = \psi \to \chi[\phi/p] \to \chi[\psi/p]$, where both ϕ and ψ are free for p in χ;

Com $\exists X \forall p(Xp \leftrightarrow \phi)$, where X is not free in ϕ;

Gen Infer $\phi \to \forall x \phi$ from $\phi \to \psi$, provided x (a propositional or operational variable) is not free in ϕ.

The conventional wisdom concerning the interactions between grounding and generalizations might be naturally strengthened as follows: a partial ground of a true generalization is either a true instance of this generalization or a partial ground of some true instance(s) of this generalization. Similar principles are suggested for conjunction and disjunction. For example in the former case, a partial ground of a true conjunctive proposition is either a conjunct of this proposition or a partial ground of some conjunct(s) of this proposition. These principles can be captured by the notion of immediate partial grounding and precisely regimented in our current language:[4]

(4) $\forall p(\phi \to \phi \prec_i \exists p \phi)$;

(5) $\forall q(q \prec_i \exists p \phi \to \exists p(q = \phi))$;

(6) $\forall pq(p \circ q \to p \prec_i (p \circ q) \circ q \prec_i (p \circ q))$;

(7) $\forall pqr(r \prec_i (p \circ q) \to r = p \vee r = q)$.

With principles (6) and (7), two connectives $\widehat{\wedge}$ and $\widehat{\vee}$ can be defined so that the following principles turn out to be deducible:[5]

(8) $\forall pq(p \mathbin{\widehat{\wedge}} q \leftrightarrow p \wedge q)$;

(9) $\forall pq(p \mathbin{\widehat{\vee}} q \leftrightarrow p \vee q)$;

(10) $\forall pqp'q'(p \wedge q \wedge (p \mathbin{\widehat{\wedge}} q) = (p' \mathbin{\widehat{\wedge}} q') \to p = p' \wedge q = q')$;

(11) $\forall pqp'q'((p \mathbin{\widehat{\vee}} q) = (p' \mathbin{\widehat{\vee}} q') \to (p \wedge \neg q \to p = p') \wedge (\neg p \wedge q \to q = q'))$.

Then consider two factive operations X and Y.[6] We suppose that $\exists p(p \mathbin{\widehat{\wedge}} (Xp \mathbin{\widehat{\vee}} \neg Xp)) = \exists p(p \mathbin{\widehat{\wedge}} (Yp \mathbin{\widehat{\vee}} \neg Yp))$. For each p, if Xp, $p \mathbin{\widehat{\wedge}} (Xp \mathbin{\widehat{\vee}} \neg Xp)$ is true by (8) and (9). Given (4), it is an immediate partial ground of $\exists p(p \mathbin{\widehat{\wedge}} (Xp \mathbin{\widehat{\vee}} \neg Xp))$, so by LL and (5), it is identical to $q \mathbin{\widehat{\wedge}} (Yq \mathbin{\widehat{\vee}} \neg Yq)$ for some q. Then, according to (8)-(11), $p = q$ and $Xp = Yq$, which means Yp. The converse direction can be proved in the same way. Thus we have:[7]

(12) $\forall XY(\Phi(X) \wedge \Phi(Y) \wedge \Psi(X) = \Psi(Y) \to X \equiv Y)$.

[4] $\circ \in \{\wedge, \vee\}$.
I omit the principles for propositional universal quantifiers since we don't need them in the following argument; but like the case of Krämer's puzzle, they are troublesome too. (I also omit principles for operational quantifiers.)
Alternatively, if we're willing to accept that there are ungrounded truths, we don't even need the principles for disjunction.

[5] Here are the definitions: suppose there are at least two truths A and B, then $\phi \mathbin{\widehat{\wedge}} \psi := ((A \wedge B) \wedge \phi) \wedge ((A \wedge A) \wedge \psi)$ and $\phi \mathbin{\widehat{\vee}} \psi := ((A \wedge B) \wedge \phi) \vee ((A \wedge A) \wedge \psi)$. The inference for (10) and (11) below will be a little tedious but not very hard.
The assumption that there is more than one truth is very modest; otherwise, either all truths ground all truths or all truths ground no truths.

[6] An operation X is factive just in case $\forall p(Xp \to p)$.

[7] We use $\Phi(X)$ to abbreviate $\forall p(Xp \to p)$, $\Psi(X)$ to abbreviate $\exists p(p \mathbin{\widehat{\wedge}} (Xp \wedge \neg Xp))$, and $X \equiv Y$ to abbreviate $\forall p(Xp \leftrightarrow Yp)$.

This looks like a principle of structurism. Indeed, there exists a Russel-Myhill argument for the inconsistency of (12) within the logic we embrace now. This argument will rely on Ref, Com and Gen.[8] So the basic idea behind Fritz's puzzle is that if grounding performs so and so, as most friends of grounding expect, then reality will become too fine-grained to be consistent.

[8] I leave the proof as an exercise. (Hint: consider $p \wedge \exists Y(\Phi(Y) \wedge p = \Psi(Y) \wedge \neg Yp)$.)
Firtz presents this puzzle in a plural logic and the principles he employs in the Russell-Myhill argument includes plural comprehension: $\exists pp \forall p(p \,\epsilon\, pp \leftrightarrow \phi)$. Boris Kment ([REF]) recently argues that some plausible principles of grounding provide us natural reason to reject plural comprehension. Thus the original version of Firtz's argument might be blocked. However, as Kment himself concedes, these principles fail to provide us natural reason to reject Com.

3. *Fritz's solution*

Ironically, Fritz ([REF]) himself puts forward a promising solution to Krämer's puzzle as well as his own one. This time, let's work in a higher-order language $\mathcal{J}$: we have singular terms—terms of type e; for any sequence of types $\langle \sigma_1, \ldots, \sigma_n \rangle$, we have predicates of type $\langle \sigma_1, \ldots, \sigma_n \rangle$ and we may call predicates of type $\langle \rangle$ formulae. For each type σ, there are countably many variables $x_1, x_2, \ldots$ and some (perhaps zero) constants $c_1, c_2, \ldots$ of this type. If M is of type $\langle \sigma_1, \ldots, \sigma_n \rangle$ and $N_1, \ldots, N_n$ are of types $\sigma_1, \ldots, \sigma_n$, then $(MN_1 \ldots N_n)$ is a formula. Finally, if ϕ is a formula and $x_1, \ldots, x_n$ are pair-wise distinct variables of types $\sigma_1, \ldots, \sigma_n$, then $(\lambda x_1 \ldots x_n.\phi)$ is a predicate of type $\langle \sigma_1, \ldots, \sigma_n \rangle$.[9]

[9] We may sometimes omit brackets when the context is clear enough.

In this language, all logical terms are treated as predicates: $\neg$ is of type $\langle\langle\rangle\rangle$ and binary connectives like $\prec$ are of type $\langle\langle\rangle, \langle\rangle\rangle$.[10] More significantly, for each σ, we have quantifiers $\forall_\sigma$ and $\exists_\sigma$ of type $\langle\langle\sigma\rangle\rangle$. So $\exists_\sigma$, for instance, combines predicates F of type $\langle\sigma\rangle$ to form formulae $\exists_\sigma F$, and every formula of the form $\exists_\sigma x\phi$ is merely an abbreviation of $\exists_\sigma(\lambda x.\phi)$ where $\lambda x.\phi$ is of type $\langle\sigma\rangle$. What's more, for each σ there is a $=_\sigma$ of type $\langle\sigma,\sigma\rangle$, defined as $\lambda xy.\forall_{\langle\sigma\rangle} X(Xx \rightarrow Xy)$.[11]

[10] For readability, we still write $\phi \prec \psi$ instead of $\prec \phi\psi$.

[11] We may sometimes omit the subscripts of quantifiers or identity signs when the context is clear enough.

Here's the smallest classical logic governing $\mathcal{J}$:[12]

[12] Ref, LL and Com now become theorems of this logic.
Here's a proof of a general version of Com ($\exists X \forall x_1 \ldots x_n(Xx_1 \ldots x_n \leftrightarrow \phi)$): Let $\bar{x}$ be $x_1 \ldots x_n$. $(\lambda\bar{x}.\phi)\bar{x} \leftrightarrow \phi$ is an instance of β_{E}. By β_{E} and Gen we get $\forall\bar{x}((\lambda\bar{x}.\phi)\bar{x} \leftrightarrow \phi)$. Since X is not free in ϕ, β_{E} and EG give us $\exists X\forall\bar{x}(X\bar{x} \leftrightarrow \phi)$.

PC All theorems of propositional calculus;

UI $\forall F \rightarrow Fa$;

EG $Fa \rightarrow \exists F$;

β_{E} $(\lambda x_1 \ldots x_n.\phi)N_1 \ldots N_n \leftrightarrow \phi[N_i/x_i]$, where each N_i is free for the corresponding x_i in ϕ;

mp Infer ψ from $\phi \rightarrow \psi$;

Gen Infer $\phi \rightarrow \forall F$ from $\phi \rightarrow Fx$, provided x is not free in ϕ;

Inst Infer $\exists F \rightarrow \phi$ from $Fx \rightarrow \phi$, provided x is not free in ϕ.

In $\mathcal{J}$, the conventional wisdom that true generalizations are grounded in their true instances should be reformulated:

(13) $\forall X \forall x(Xx \rightarrow Xx \prec \exists X)$;

(14) $\forall X(\forall X \to \forall x(Xx \prec \forall X))$.

Note that (13) is not (1). To get (1) from (13) and trigger the previous argument, we need an auxiliary principle:[13]

$\beta_=$ $(\lambda x_1 \ldots x_n.\phi)N_1 \ldots N_n = \phi[N_i/x_i]$.

Alternatively, if true λ-applications are always grounded in their β-reductions and if grounding is transitive, (13) entails (1):

β_G $\phi[N_i/x_i] \to \phi[N_i/x_i] \prec (\lambda x_1 \ldots x_n.\phi)N_1 \ldots N_n$;

Tr $\forall pqr(p \prec q \land q \prec r \to p \prec r)$.

But why should we accept $\beta_=$ or β_G? It seems rejecting them is a relatively easier option for those grounding theorists—at least easier than rejecting other logical or ground-theoretical principles.[14]

The lesson: By adopting a more expressive language of metaphysics, we are committed to more structures in reality. Once some originally intertwisted structures are disentangled, more theoretical options may turn out to be accessible.[15]

But questions remain:

Q1 Could we end up with a consistent picture if we merely reject $\beta_=$ and β_G?

Q2 Do we have sufficient philosophical reasons to reject them?

Jeremy Goodman ([REF]) does give an affirmative answer to the first question and tries to give an affirmative answer to the second. In what follows, we will reconstruct his first answer and evaluate his second answer.

4. *The hierarchy of propositions*

We start with an abstract mathematical structure $(S, \mathtt{c}, \mathtt{d})$: S is a non-empty set and $\mathtt{c}, \mathtt{d}$ are two objects not in S. We then build a series of models as follows:

- $P_0 = S$;
- $P_{\alpha+1} = P_0 \cup \{\{\mathtt{c}\} \cup X : X \subseteq P_\alpha\} \cup \{\{\mathtt{d}\} \cup X : X \subseteq P_\alpha\}$;
- $P_\alpha = \bigcup_{\beta<\alpha} P_\beta$ if α is a limit ordinal.

The most basic ideas behind nearly all pictures developed by Goodman:

- Propositions are isomorphic to P_γ for some limit ordinal γ;[16]
- $\mathtt{c}$ corresponds to the structure of conjunction and $\mathtt{d}$ corresponds to the structure of disjunction.

[13] In fact, $(\lambda p.\phi)\psi = \phi[\psi/p]$ will be enough.
Proof: An instance of (13) is $(\lambda p.\phi)p \to (\lambda p.\phi)p \prec \exists p\phi$. So by using $\beta_=$ twice, we can get $\phi \to \phi \prec \exists p\phi$, with the help of LL. Then UI and $\beta_=$ will give us its universal closure.

[14] Firtz's puzzle can be solved in a similar way. With neither $\beta_=$ nor β_G, it is not clear whether $p \mathbin{\widehat{\land}} (Xp \mathbin{\widehat{\lor}} \neg Xp)$ is still a ground, immediate or mediate, of $\exists(\lambda p.(p \mathbin{\widehat{\land}} (Xp \mathbin{\widehat{\lor}} \neg Xp)))$.

[15] We originally recognized one structure of quantification. But in the higher-order setting, we have a somehow different structure of quantification (now it becomes a specific case of the structure of predication) as well as a structure of λ-abstraction.

[16] We will encounter a different model in §?.

Such a P_γ is in essence an algebra. The remaining task is to coherently 'embed' other structures of reality (predication, λ-abstraction, partial grounding, etc.) in this algebraic structure, keeping the validity of those logical and ground-theoretical principles at the same time.[17]

Before we turn to more concrete pictures, we need an important notion—the *levels* of propositions. Suppose i is the isomorphism from propositions to P_γ. For each proposition ϕ, the level of ϕ, $\mathsf{l}(\phi)$, is α $(< \gamma)$ if $\mathsf{i}(\phi) \in P_\alpha$ and $\mathsf{i}(p) \notin P_{\alpha-1}$.[18]

[17] It might be helpful to think about Classicism. The idea is similar: propositions form a Boolean algebra under truth-functional operations, and other structures can be coherently 'embedded' in it.

[18] Since no ordinal is less than 0, all $\mathsf{i}(\phi)$ are not in P_{0-1}.

5. *Goodman's first picture*

We assign a unique *rank* to each type:

- If σ is not monadic, $\mathsf{r}(\sigma) = \alpha_\sigma$ for some α_σ;
- $\mathsf{r}(\langle\sigma\rangle) = \mathsf{r}(\sigma) + \alpha_{\langle\sigma\rangle}$ for some $\alpha_{\langle\sigma\rangle} \geq 1$.

Then the general structure of predication is subject to the following constraint:

($\star$) If F is a property of type $\langle\sigma\rangle$ and a is an entity of type σ, then $\mathsf{l}(Fa) \leq \mathsf{r}(\sigma)$; in particular, when $F = \lambda x.\phi$,

$$(\lambda x.\phi)a = \begin{cases} \phi[a/x] & \text{if } \mathsf{l}(\phi[a/x]) \leq \mathsf{r}(\sigma) \\ \phi^* \text{ where } \Box(\phi^* \leftrightarrow \phi) \text{ and } \mathsf{l}(\phi^*) \leq \mathsf{r}(\sigma) & \text{otherwise} \end{cases}$$

$\Box$ is the broadest necessity. Here, we may define it as $\lambda p.((\lambda q.q)p = (\lambda q.q)(p \to p))$. Given Goodman's model theory, it is an S5 operation.[19]

We also need some more specific embeddings (recall the isomorphism i):

- $\mathsf{i}(\phi \wedge \psi) = \{\mathsf{c}, \mathsf{i}(\phi), \mathsf{i}(\psi)\}$;
- $\mathsf{i}(\phi \vee \psi) = \{\mathsf{d}, \mathsf{i}(\phi), \mathsf{i}(\psi)\}$;
- $\mathsf{i}(\phi \prec \psi) = \{\mathsf{d}\} \cup \{\{\mathsf{c}, r_1, ..., r_n\} : \mathsf{i}(\phi) = r_1 \in ... \in r_n = \mathsf{i}(\psi) \neq \mathsf{i}(\phi)\}$;
- $\mathsf{i}(\forall_\sigma F) = \{\mathsf{c}\} \cup \{\mathsf{i}(Fx) : \text{for all } x \text{ of type } \sigma\}$;
- $\mathsf{i}(\exists_\sigma F) = \{\mathsf{d}\} \cup \{\mathsf{i}(Fx) : \text{for all } x \text{ of type } \sigma\}$.

[19] Some audiences may feel if we define $\Box$ in terms of λ, ($\star$) is circular. But ($\star$) is just a constraint, not a definition. The legitimate question is whether this constraint can be satisfied. And Goodman shows it can.

Now, let's turn back to Krämer's puzzle. Recall that $\exists p\, p$ is just an abbreviation of $\exists(\lambda p.p)$. $\mathsf{i}((\lambda p.p)\exists(\lambda p.p)) \in \mathsf{i}(\exists(\lambda p.p))$ and therefore the level of $\exists(\lambda p.p)$ is strictly greater than the level of $(\lambda p.p)\exists(\lambda p.p)$—this is allowed according to ($\star$): the rank of $\exists$ is strictly greater than $\lambda p.p$. Thus, two consequences: one, $(\lambda p.p)\exists(\lambda p.p)$ is an (immediate) partial ground of $\exists(\lambda p.p)$; two, both $\beta_=$ and β_G are false. It is not hard to verify that (3), (13), (14), and the logic of $\mathcal{J}$ hold in this picture. So Q1 is answered. And Q2 is transferred to this question: do we have sufficient philosophical reason to accept Goodman's picture?[20]

[20] In fact, most higher-order metaphysicians, including Goodman himself ([REF]), tend to accept $\beta_=$. The most serious argument for $\beta_=$ I have ever seen comes from Andrew Bacon. In [REF], he argues that we have theoretical pressure to pin down the meaning of those λ-terms uniquely. The most straightforward (and perhaps the most natural) way to so is to adopt the principle of $\beta\eta$-conversion which entails $\beta_=$. I point out in [REF] that it is possible to develop a comprehensive theory of grounding without $\beta_=$ (or β_G), in which the meaning of those λ-terms is uniquely pinned down by other principles of granularity. It seems Goodman has no similar resources in his theory. However, I don't think this is a compulsory requirement, especially for friends of grounding. So I won't say too much about $\beta_=$ below.

Objections to ($\star$)

Call a property or a relation F of type $\langle\sigma_1, ..., \sigma_n\rangle$ *bounded* if there is an α such that $\mathsf{l}(Fx_1...x_n) \leq \alpha$ for all $x_1, ..., x_n$ of type $\sigma_1, ..., \sigma_n$; a property or a relation is unbounded if it is not bounded.

We consider three objections in turn:

(a) Negation

(b) The monadic/polyadic distinction

(c) The distribution of properties and relations

A revenge argument

In $\mathcal{J}$, we have quantifiers for unary predicates. Someone may suggest for each $n > 0$, we should also have some corresponding quantifiers combining n-ary predicates to form formulae. If so, consider this generalization of (13):

(15) $\forall X^{\langle\sigma_1,...,\sigma_n\rangle}\forall x_1^{\sigma_1}...x_n^{\sigma_n}(Xx_1...x_n \to Xx_1...x_n \prec \exists_{\sigma_1,...,\sigma_n}X)$.

Here's an instance of (15):

(16) $\forall X \forall pq(Xpq \to Xpq \prec \exists_{\langle\rangle,\langle\rangle}X)$.

But (16) is in conflict with the law that true conjunctions are partially grounded in their conjuncts.[21]

[21] Proof: Let ϕ be a truth. By UI, (16) implies $\phi \wedge (\exists_{\langle\rangle,\langle\rangle}\wedge) \to \phi \wedge (\exists_{\langle\rangle,\langle\rangle}\wedge) \prec \exists_{\langle\rangle,\langle\rangle}\wedge$. Since there are true conjunctions, $\phi \wedge (\exists_{\langle\rangle,\langle\rangle}\wedge) \prec \exists_{\langle\rangle,\langle\rangle}\wedge$. But the former should also be grounded in the latter.

6. *Revising the first picture*

If we have quantifiers for polyadic predicates and similar principles of grounding governing them, we need some revisions. Goodman suggests a *syncategorematic* treatment of conjunction and disjunction. And a similar strategy can be adopted for negation as well. So $\neg$, $\wedge$ and $\vee$ are not predicates now; instead, they are untyped expressions like λ, associated with certain rules of term-formation:

- If ϕ is of type $\langle\rangle$, then $\neg\phi$ is of type $\langle\rangle$;
- If ϕ and ψ are of type $\langle\rangle$, then $\phi \circ \psi$ is of type $\langle\rangle$ ($\circ \in \{\wedge, \vee\}$).

The puzzle in the end of the last section is solved.

I have one more suggestion. Recall the model P_γ of propositions. Let $\gamma > \omega$ and $\mathsf{r}(\sigma) = \omega$ if σ is non-monadic. Therefore, $\mathsf{r}(\langle\sigma\rangle) > \omega$ for all σ. But let's impose the following constraint:

($*$) If F is a property of type $\langle\sigma\rangle$, which is neither $\forall$ nor $\exists$ of the same type, then for all x of type σ, $\mathsf{l}(Fx) \leq \omega$.

According to this picture, the distribution of properties and relations within level ω obeys no restrictions, and it seems we therefore receive enough degree of freedom to do systematic metaphysics with a robust notion of grounding. Beyond ω, the remaining structure of predication is just the structure of quantification—a relatively 'boring' structure. Such an additional 'patch' seems to be a reasonable cost.

7. *Some deeper problems?*

Truth, knowledge, and belief

Consider the four principles below:

(T) $\forall p(p \to p \prec T\ulcorner p\urcorner)$;[22]

(K) $\forall x\forall p(Kxp \to p \prec Kxp)$;

(B_1) $\forall x\forall p(Kxp \to p \prec Bxp)$;

(B_2) $\forall x\forall p\forall e(\text{has}(x,e) \wedge \text{that}(e,p) \wedge \text{know}(e) \to p \prec \text{has}(x,e))$.

[22] We can take $\ulcorner\cdot\urcorner$ as a syncategorematic expression like λ and truth-functional connectives. Fine's original puzzle relies on (T) or something near enough.

If one of these principles turns out to be true, we can also run a revenge argument. However it seems a grounding theorist indeed has good philosophical reasons to accept at least one of them.

Replies: (T) and (K) need to be revised. In the current setting, a proposition and it's being true may not be the same matter. I tend to think it is propositional truth ($(\lambda p.p)p$) that grounds sentential truth ($T\ulcorner p\urcorner$).[23] And I also tend to think it is a proposition's being true, rather than the proposition itself, that partially constitutes the proposition's being known. So I suggest the correct principles are:[24]

(T′) $\forall p(p \to (\lambda p.p)p \prec T\ulcorner p\urcorner)$;

(K′) $\forall x\forall p(Kxp \to (\lambda p.p)p \prec Kxp)$.

[23] Goodman sharply points out given a classical semantics for negation, (T) implies the unrestricted T-schema: $\forall p(T\ulcorner p\urcorner \leftrightarrow p)$, which is classically inconsistent.

[24] Given β_G, which we don't have, (T) and (K) are entailed by (T′) and (K′).
Goodman says:"the more p becomes alienated from $[(\lambda p.p)p]$ in its grounding behavior, the less plausible it is that the latter as opposed to the former is the appropriate ground of our knowledge of the former" (p. 22). I don't know why.

(B_1) and (B_2) need to be revised too. To my understanding, it is the true proposition p itself (or the corresponding fact) that 'grounds' Bxp according to B_1 and $\text{has}(x,e)$ according to B_2. But here, the connection at issue is more probably a causal relation, not a metaphysical grounding.

Grounding and fundamentality

It's the time to rethink the tenet that no proposition grounds itself, namely (3).[25] Why must we accept it? What the answer looks like

[25] My argument in this subsection is inspired by Ted Sider's famous argument against ungrounded grounding truths. See, for example, [REF], pp. 748-749.

and whether it seems plausible depends, I believe, on our understanding of grounding. Well, even though we make some further clarifications of what we talk about when we talk about ground, I feel the answer will be still vague to a great extent. But let's have a try.

My favorite strategy is to regard grounding as a kind of connection between the fundamental and the non-fundamental.[26] Since it is nearly a conceptual truth that nothing is *less* fundamental than itself, it follows that nothing is self-grounded. To make this idea more precise, we may enrich our language with a type-$\langle\sigma,\sigma\rangle$ predicate $\leq^{\sigma}_{\mathrm{F}}$, for each σ, denoting the *as fundamental as* relation of type-σ entities. Then, two principles:

Grounding the Less Fundamental $\forall pq(p \prec q \to \neg(q \leq^{\langle\rangle}_{\mathrm{F}} p))$;

As Fundamental as Itself $\forall_{\sigma} x(x \leq^{\sigma}_{\mathrm{F}} x)$.

(3) is obviously a consequence of them.

Now, for the sake of argument, consider a particular instance of As Fundamental as Itself:

(17) $\exists F \leq^{\langle\rangle}_{\mathrm{F}} \exists F$,

where F is a propositional operation. Given (17), should we accept (18) below on the same ground?

(18) $\exists F \leq^{\langle\rangle}_{\mathrm{F}} F(\exists F)$.

According to Goodman's picture, we have

(19) $F(\exists F) \prec \exists F$,

no matter what F is. If (18), like (17), is in fact true, a similar argument, like our previous argument against the cases of self-grounding, can be run. Goodman will then find himself in an embarrassing situation. He tries to save a principle within his picture, but the ground of our best reason to save this principle also provides us reason to rule out his picture.

The truth of (17) is robust because it is invariant under different conceptions of fundamentality. No doubt (18) can't be as robust as (17). But it needs not. As long as it makes sense to endorse (18) according to the conception of fundamentality widely shared by those grounding theorists, my aim is achieved. Unfortunately, it is really hard to articulate such a conception of fundamentality. To illustrate my argumentative strategy, let's start with several clearer ones.

First, by the structuralist conception of fundamentality, being fundamental is being simple. Thus, the degree of fundamentality is negatively correlated to the degree of complexity. Under this conception, we have $\exists F \leq^{\langle\rangle}_{\mathrm{F}} \exists F$, as a limit case, and also $\exists F \leq^{\langle\rangle}_{\mathrm{F}} F(\exists F)$

[26] Someone may think grounding is a kind of worldly dependence. If nothing else is added however, I'm not sure whether grounding must be irreflexive. Perhaps, for instance, the existence of God is self-dependent. Many fans of grounding are inclined to take it as a kind of metaphysical explanation. And they emphasize, again and again, although explanation is an epistemic notion, metaphysical explanation is not. This sounds like "a white horse is not a horse". To be charitable, I can at best understand their point in this way: grounding is a kind of worldly dependence sharing some significant formal features with explanation. Therefore, if you tend to think that noting explains itself, you may also hope to insist that nothing grounds itself. But it is not clear enough to me whether explanation must be irreflexive either.

because the latter is more complex than the former. Of course, the structuralist idea might be problematic due to the Russell-Myhill.[27]

[27] Many friends of grounding embrace a structuralist picture without reflection. But there are indeed consistent structuralist pictures. As Goodman teaches us, even the most naive idea of structuralism holds for entities of recoverable types. A more sophisticated version of structuralism for all entities is developed by Bacon ([REF]).

Even though a structured world is too fine-grained, *aboutness* may still be a legitimate notion. If so, the following principle seems attractive:

Sider WBW 148

Combination $\forall_\sigma xy(y$ is about everything that x is about $\to x \leq^\sigma_{\mathrm{F}} y)$.

A mature theory of aboutness is still wanting,[28] and, to be honest, I really don't know how to regiment "y is about everything that x is about" formally—the difficulty is to regiment "everything" in a finite length. But an intuitive grasp of this notion is enough. Again, notice that (17) (and As Fundamental as Itself) are entailed by Combination provided that everything is about everything that itself is about. It is also intuitive that $\exists F$ is not about more things than $F(\exists F)$, so (18) is true. Of course, not all grounding theorists are willing to accept all instances of Combination.[29]

[28] See Dorr et al. [REF], §15.3 for a basic theory of aboutness. What we need here, I guess, should be a more comprehensive theory.

[29] Intuitively, $\neg\phi$ is about everything that $\neg\neg\neg\phi$ is about. So by Combination, $\neg\neg\neg\phi \leq^{\langle\rangle}_{\mathrm{F}} \neg\phi$. But for many guys, $\forall p(p \to p \prec \neg\neg p)$, though I myself tend to accept $\forall p(p = \neg\neg p)$.

Grounding theorists are inclined to understand fundamentality in terms of grounding. One basic idea is that grounding introduces some new non-propositional entities which are non-fundamental. For example, ϕ and ψ jointly ground their conjunction $\phi \wedge \psi$. $\wedge$ is somehow *new* for both ϕ and ψ even though ϕ or ψ may involve $\wedge$. Clearly, a proposition has nothing new for itself. And it is hard to say that $\exists F$ has something new for $F(\exists F)$.

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