Linnebo's "Generality explained"

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(1) Every student of mine was born on a Monday.
(2) Every whale is a mammal.

An explanation for (1):

- My students are $a_1, \ldots, a_m$, and a_1 was born on a Monday,…,a_m was born on a Monday.
  - This is an (partially) instance-based explanation of (1).

Some explanations of (2):

- It's essential to whale-hood that every whale is a mammal.
- Part of what it is to be a whale is to be a mammal. ($\exists P(W \equiv \lambda x\,.\,(Mx \wedge Px))$)
- It's a consequence of some non-Humean laws of nature that every whale is a mammal.
  - These are (purely) generic explanations of (2).

(To say that (2) admits of generic explanation is not to deny that it may also have instance-based based explanation, perhaps by listing all the whales there are and note that each is a mammal.)

The proposed explanation of (1) is not *purely* instance based, because the universal quantifier in (1) may well be unrestricted in scope, but its explanation doesn't mention everything.

Linnebo has three main aims in this paper

- To develop a truthmaker semantics for universal generalizations which would hopefully shed light on how instance-based and generic explanations work.
- To show that, where the domain doesn't have a definite range of instances, instance-based explanations are not always available.
- To show that, when instance-based explanations are available, the logic of the quantifiers is classical. When instance-based explanations are not available, intuitionistic logic remains valid.

1. Why generic explanations are needed

Linnebo gives some examples where a universal generalization has only generic explanations.

- If the future is metaphysically open, it's indeterminate what whales will come into existence. So it's impossible to list all the instances of (1). So on such a view, (1) doesn't have a purely generic explanation

- According to set-theoretic potentialism, there's no totality of all sets. If so, it's impossible to consider all sets. So there's no purely instance-based explanation of the propositions like *every set has a power set.*

- Suppose that what exists depends on "which concepts we bring to bear in our thoughts and theories", such that there's no "definite totality of absolutely all objects". Then, presumably, there's no purely instance-based explanation of absolutely universal generalizations.

- Here's a way of articulating the "hierarchical conception of reality". Each truth is assigned a level, k, which should by fully explained by truths of levels $< k$. Consider a universal generalization $\forall x \phi(x)$, of level i. One of its instances would be $\phi(\forall x \phi(x) \wedge \psi)$, which would presumably be of level $j > i$. So some instance of $\forall x \phi(x)$ is of a higher-level, but $\forall x \phi(x)$ should be explained by truths of lower-levels. So $\forall x \phi(x)$ can't be explained by all of its instances.

- If mathematical truths are explained by finite proofs, then no universal generalization over infinite domains has a wholly instance-based explanation.

2. Intrinsic truthmaking

The central notion of Linnebo's truthmaker semantics is that of a *state s* verifying a formula ϕ relative to a variable assignment σ: $s \Vdash \phi(a_1, \ldots, a_m)$, where $\sigma(v_i) = a_i$.

We have a set of states S, a partial order $\leq$ on S, and a fusion operation $\sqcup$ on S. We can think of $\leq$ as saying one state's being less or equally informative than another. There's a bottom state $\mathbf{0}$ of the ordering $\leq$, which we can think of as having no informational content. And there's an inconsistent top state $\mathbf{1}$, which verifies all statements.

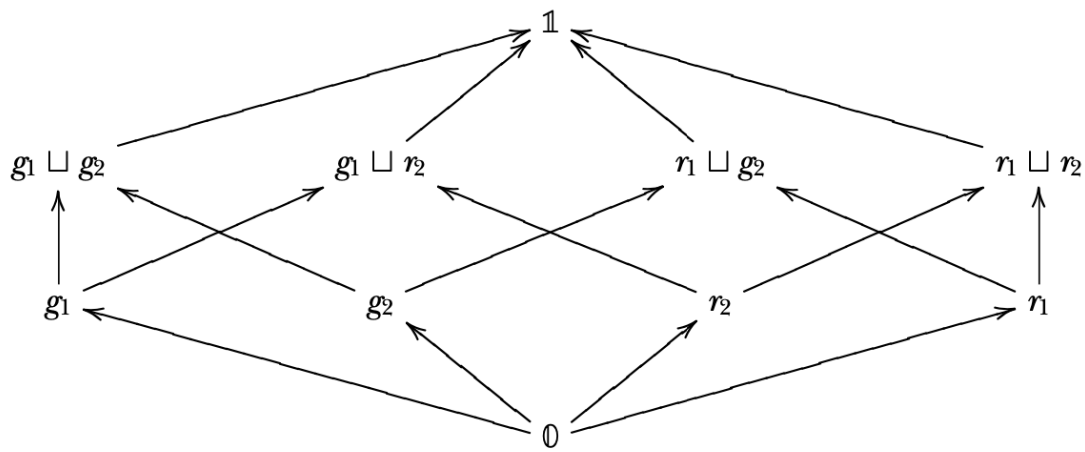
One distinctive feature of Linnebo's version of truthmaker semantics is that it is *inexact*. The notion of verification of his account is *monotonic*, in the following sense:

If $s_1 \Vdash \phi$ and $s_1 \leq s_2$, then $s_2 \Vdash \phi$.

Here's how Linnebo motivates the monotonic character of his semantics: "My target idea is that a state *s* verifies a statement φ just in case *material intrinsic to s suffices to explain φ*, leaving no need to "look beyond" *s* to account for the truth of φ. The truth of φ is in this sense intrinsic to *s*." (p.357)

A state s is *atomic* iff $\forall t(t < s \rightarrow t = \mathbf{0})$.
A state s is *maximally consistent* iff $\forall t(s < t \rightarrow t = \mathbf{1})$.

Example 1.
Consider a system consisting of two balls, b_1 and b_2, each of which can be either red or green. We thus have four atomic states, which we designate r_1, r_2, g_1, and g_2. The maximal consistent states are $r_1 \sqcup r_2$, $r_1 \sqcup g_2$, $g_1 \sqcup r_2$, and $g_1 \sqcup g_2$. Other fusions of distinct atomic states are inconsistent: $r_1 \sqcup g_1 = r_2 \sqcup g_2 = 1$.



Holding the domain fixed, what verifies $\forall x Rx$? A natural answer is: $r_1 \sqcup r_2 \Vdash \forall x Rx$. This is an example of instance-based verification.

Example 3. Consider a system consisting of a countable infinity of balls, b_i for $i \in \omega$, each of which can be green or either of two non-overlapping shades of red, namely crimson and scarlet. The atomic states are g_i and r_i for each i. The maximal consistent states are obtained by choosing one of g_i, c_i, and s_i, for each i, and fusing all the chosen states.

What verifies $\forall x(Cx \rightarrow Rx)$? This seems to require no information about what the colors of the balls are. So a natural answer is: $\mathbf{0} \Vdash \forall x(Cx \rightarrow Rx)$.

To systematize our judgments about these particular cases, let's now lay out semantic clauses for logically complex formulas.

$s \Vdash \phi \wedge \psi$ iff $s \Vdash \phi$ and $s \Vdash \psi$.
$s \Vdash \phi \vee \psi$ iff $s \Vdash \phi$ or $s \Vdash \psi$.
$s \Vdash \phi \rightarrow \psi$ iff for each t, if $t \Vdash \phi$ then $t \sqcup s \Vdash \psi$.

- For example, in Example 1, $r_2 \Vdash Rb_1 \rightarrow (Rb_1 \wedge Rb_2)$, because $r_1 \sqcup r_2 \Vdash Rb_1 \wedge Rb_2$. Also, $g_1 \Vdash Rb_1 \rightarrow (Rb_1 \wedge Rb_2)$, because $g_1 \sqcup r_1 = \mathbf{1} \Vdash Rb_1 \wedge Rb_2$.

$s \Vdash \neg\phi$ iff $\forall t(t \Vdash \phi \rightarrow t \sqcup s \Vdash \bot)$.

- We might say that s excludes ϕ iff the fusion of s with any state that verifies ϕ is the inconsistent state. Thus, the clause says that $s \Vdash \neg\phi$ iff s excludes ϕ.

For the semantic clauses for quantifiers, we associate each state s with a domain $D(s)$. Intuitively, $D(s)$ is the set of objects that s is about. For example, $D(g_1)$ in Example 1 is b_1.

$s \Vdash \exists x\phi(x)$ iff $s \Vdash \phi(a)$, for some $a \in D(s)$
$s \Vdash \forall x\phi(x)$ iff $s \sqcup t \Vdash \phi(a)$, for every t and every $a \in D(t)$.

"It would be unreasonable to require that, for a state s to verify a universal generalization $\forall x$ $\varphi(x)$, the state s verify each instance $\varphi(a)$ completely on its own. Since s may "know" nothing about a, we may need information about what object a is." (p.362-363)

3. Instance-based versus generic verification

Recall the opening example (1): Every student of mine was born on a Monday. Suppose that $s_i \Vdash Ma_i$. Note that $s_1 \sqcup \ldots \sqcup s_m$ arguably doesn't verify (1), since I could have had more students. So we also need a totality state $t \Vdash \forall x(Sx \rightarrow (x = a_1 \vee \ldots \vee x = a_m))$.

Is purely instance based verification possible? That is, is there a state s that verifies $\forall x\phi(x)$ without involving any sort of generic verification or totality states?

- The answer would be negative, given this assumption: for any state s there is an extension $s' \geq s$ with a strictly larger domain. Given this assumption, if $s \Vdash \forall x\phi(x)$, s must verify that some objects that s is *not* about is also ϕ, which means that the verification must be partially generic or involve totality states.

Some characteristics of instance-based and generic verification
Instance-based:

- Low uniformity: the verifier can be "factorized" as a fusion of totality states and particular states.
- High aboutness: the verifier is about many objects.
- Low modal robustness: supposing that $s = s_1 \sqcup \ldots \sqcup s_m \sqcup t$, then given a "larger" totality state T, $s_1 \sqcup \ldots \sqcup s_m \sqcup T$ could fail to verify the universal generalization.

Generic:

- High uniformity: the verifier doesn't contain totality states and can't be factorized into simpler states.
- Low aboutness: the verifier need not be about any particular objects.
- High modal robustness: the verifier could still verify the universal generalization were there to be more objects.

Which true universal generalizations admit of which type of explanation?

- Since generic explanations are uniform across all instances, it's unavailable for merely accidental universal generalizations.
- If there is not a definite domain of all objects, there is not a definite range of all instances to consider for the purposes of giving an entirely instance-based explanation. If there's no definite range of all objects, then there's no state that is about all objects. But such a state is required for purely instance-based verification.

4. The logic of quantification

We can have two intuitive notions of logical truths. The first one says that logical truths are those verified by the trivial state $\mathbf{0}$. The second says that logical truths are those verified by each maximally consistent state (≈ true in every possible world).

The two notions of logical truths correspond to two consequence relations:

Definition 1.

(a) Let $\Sigma \vDash \phi$ iff: for every state space S and every state $s \in S$, if s verifies every member of Σ, then s verifies ϕ as well.

(b) Let $\Sigma \vDash^* \phi$ mean that, for every state space S and every *maximal consistent* $s \in S$, if s verifies every member of Σ, then s verifies ϕ as well.

These two consequence relations correspond to intuitionistic and classical deducibility, respectively:

Proposition 1. Consider the language of first-order logic. Let $\vdash_{IL}$ and $\vdash_{CL}$ represent deducibility in intuitionistic and classical logic, respectively. Then:

(a) $\Sigma \vdash_{IL} \phi$ iff $\Sigma \vDash \phi$. In particular, $\vdash_{IL} \phi$ iff $\mathbf{0} \Vdash \phi$ for every sentence ϕ and every state space S.

(b) $\Sigma \vdash_{CL} \phi$ iff $\Sigma \vDash^* \phi$. In particular, $\vdash_{CL} \phi$ iff $s \Vdash \phi$ for every sentence ϕ and every state space S and every maximal consistent $s \in S$.

Now, suppose that there's no definite domain of all objects, such that purely instance-based explanation is unavailable. Then maximal consistent states are not always available, since the fusion operation must be restricted to states that are simultaneously available. So, when purely instance-based explanation is unavailable, we can't always have classical logic, but intuitionistic logic remains valid.

5. Semi-intuitionistic logic

In the final section, Linnebo describes a "semi-intuitionistic" logic, which is a result of adding the following two principles to intuitionistic logic:

(BOM) $\forall yy((\forall x \prec yy)(\phi(x) \vee \neg\phi(x)) \rightarrow (\forall x \prec yy)\phi(x) \vee (\exists x \prec yy)\neg\phi(x))$

This principle ensures that quantification restricted to a plurality behaves classically, and admits of instance-based explanation. This turns out to be a logical truth in the sense of being verified by $\mathbf{0}$.

(At-LEM) $\forall \bar{x}(P\bar{x} \vee \neg P\bar{x})$, where Ps are atomic predicates of the language.

This principle says that the only source of non-classical behavior are the quantifiers.