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Fitch's paradox and truthmaking maximalism

Is there a purely logical argument for Maximalism?

In this presentation we shall first lay the background, explaining what is Fitch's paradox and what is the basic ideas of truthmaking. Then we recapitulate the discussion between Jago and Trueman about using Fitch-style reasoning to argue for truthmaking maximalism, and identify Jago's expectation of his argument. Finally, I will give a counter-example to show that Jago's response to Trueman is not convincing, and try to convince the audience that truthmaking maximalism should not be defended from a purely logical point of view.

1.Fitch's paradox

Frederic Fitch(1963) argues that, give certain principles, we could conclude the theorem 5 "If there is some true proposition which nobody knows (or has known or will know) to be true, then there is a true proposition which nobody can know to be true."

Theorem 5 $\exists p(p \wedge \neg Kp) \vdash \exists p(p \wedge \neg \Diamond Kp)$

The contraposition of this theorem is the famous knowability paradox, which states all possible known truths are in fact known.

Knowability paradox $\forall p(p \rightarrow \Diamond Kp) \vdash \forall p(p \rightarrow Kp)$

The knowability paradox has a bizarre result that blurs the distinction between the knowability principle with the omniscience principle. What is more, this paradox threatens many anti-realism views which hold that all truths are knowable and we could expend our

knowledge to cover more truths. The derivation of this paradox, however, is a simple inference based on rather uncontroversial premises including the following:

Conjunction distribution of knowledge(CDK) $K(p \wedge q) \vdash Kp \wedge Kq$

Factivity of knowledge(FK) $Kp \vdash p$

Necessitation(Nec) $p \vdash \Box p$

Necessity/Possibility interchangeability(NPI) $\Box \neg p \vdash \neg \Diamond p$

With these principles in place, we start with the knowability principle that all truths are possibly known $\forall p(p \rightarrow \Diamond Kp)$:

1. $\forall p(p \rightarrow \Diamond Kp)$	Knowability
2. $\exists p(p \wedge \neg Kp)$	Non-omniscient
3. $(p \wedge \neg Kp)$	An instance of Non-omniscient
4. $(p \wedge \neg Kp) \rightarrow \Diamond K(p \wedge \neg Kp)$	1,3
5. $K(p \wedge \neg Kp)$	Assumption for reductio
6. $Kp \wedge K\neg Kp$	5, CDK
7. $Kp \wedge \neg Kp$	6, FK
8. $\neg K(p \wedge \neg Kp)$	7, contradiction
9. $\Box \neg K(p \wedge \neg Kp)$	8, Nec
10. $\neg \Diamond K(p \wedge \neg Kp)$	9, NPI
11. $\neg(p \wedge \neg Kp)$	4, 10, modus tollens
12. $\neg \exists p(p \wedge \neg Kp)$	3, 11, contradiction
13. $\forall p(p \rightarrow Kp)$	

Here we get the paradox, for any truth we have already known it.

The solutions to Fitch's paradox could be classified into three kinds, logical revisions that utilize alternative logics, semantic restrictions that limit the scope of quantification in knowability principle, and the syntactic restriction to require the proposition in knowability principle to have certain logical property.[1] Though these solutions are interesting approaches, we shall not dive into the complex analyses about Fitch's paradox *per se*. Instead, this presentation will take Fitch-style reasoning as acceptable and focus on the legitimacy of applying it to the truthmaking maximalism argument.

[1] A detailed introduction of these strategies is on the SEP entry (Brogaard and Salerno, 2019)

2.Truthmaking maximalism

Truthmaking theory is built on the very plausible intuition that a truth is true in virtue of some existence - the truthmaker. We characterize this "in virtue of" relation as truthmaking, the true proposition p is made true by some existence x.[2] The first monography on truthmaking, brought by D.M.Armstrong in 2004, has maximalism and necessitarianism as two main theses for truthmaking. Afterward, these two truthmaking theses have been the main attraction of truthmaking and have been under heavy debate.

Maximalism demands that for every truth there is a truthmaker for it, which clarifies the scope of application for truthmaking encloses the set of all truths. Armstrong(2004, 7) doesn't provide an argument for maximalism, but he hopes that "philosophers of realist inclinations will be immediately attracted to the idea…" Motivated by the "realist inclinations", maximalism has not received any positive argument, not at least to my knowledge, until the Jago(2019) paper claims to do so. However, various attacks on maximalism have been raised because of the infructuous effort of trying to find truthmakers for negative truths and necessary truths. It is an easy escape to deny truthmakers for negative truths,[3] but with a heavy cost of rejecting maximalism. The hardship in finding these truthmakers is intertwined with the other thesis of truthmaking, necessitarianism.

Necessitarianism demands that if x is a truthmaker for p, then necessarily if x exists then p is true. Truthmaking theorists have reached a consensus that something could be a truthmaker for p only if certain constraints are met and cash out the "in virtue of" intuition. Necessitarianism, believed by many truthmaking theorists, is a minimum constraint — the existence of a truthmaker should necessitate the truth. Further constraints include strengthening the necessitation to relevant necessitation — the existence of truthmaker should relevantly necessitate the truth, requiring the truth is about the truthmaker — the truthmaker specifies a way for the truth to be true. However, all the existing proposals have to compromise facing the problem of negative truths.

The truthmaker for negative truths, in comparison with positive truths, is hard to come by.

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[2] Notice that a truth p could have more than one pertinent truthmakers, and a truthmaker could make serval truths true. For instance, "there is a dog" is made true by a bulldog as well as a sheep dog, and a sheep dog could make "there is a dog" as well as "there is an animal" true. So truthmaking could be a many-to-many relation.

[3] Truthmakers for necessary truths are also hard to find. Since necessary truths are true in every possible world, any existence is not central to their being true and thus fails to satisfy the "in virtue of" intuition. Many theorists agree that necessary truths stand alone for their special nature, while negative truths are mundane truths that truthmaking theory must handle. For this reason I shall concentrate on the more pressing problem of negative truths.

Take a negative existential to show this, "there is no unicorn" is true in the actual world but nothing in existence makes this proposition true. Any actual thing could co-exist with a unicorn in another possible world and fails to be the truthmaker according to the aforementioned necessitarianism. No consensus has been reached on the solutions to this problem for all the attempts have some defects (Jago (2019, 41) provides a glimpse on these solutions).

In the next section, Jago's Fitch-style argument for maximalism is showed. He basically operates with Necessitarianism at the start, and strengthens Necessitarianism with relevance constraint when responding to Trueman's objection. A caveat is due here, the truthmaking under this discussion is the metaphysical truthmaking — a cross-categorical relation between existences and propositions, not the semantic truthmaking — a logical characterization of a hyper-intensional relation. Though the latter could have effects on the former, they are not the same. Though the method is Fitch-style logical reasoning, the goal of Jago is to provide an argument for the metaphysical thesis that every truth has a truthmaker.

3.Jago vs. Trueman

The argument for maximalism by Jago(2019, 42) is simple, just swaps the knowledge operator in the argument with a truthmaking operator "TM" which means "there is a truthmaker for …" throughout Fitch paradox. The CDK principle and FK principle change into the CDTM and FTM:

Conjunction distribution of truthmaking (CDTM) $TM(p \wedge q) \vdash TMp \wedge TMq$

Factivity of truthmaking (FTM) $TMp \vdash p$

Parallel to knowability Jago argues that for every truth it is logically possible that it has a truthmaker:

Possibility of truthmaking (PT) $\forall p(p \rightarrow \Diamond TMp)$

CDTM is the uncontroversial idea that if p and q together have a truthmaker, then p has a truthmaker as well as q (the TM operator contains a quantification over truthmakers such that the truthmaker for p needs not to be the same for q). FTM is also easy to accept, where a truthmaker exists the proposition is true. The only hard sell is PT, Jago stresses that the possibility of p having a truthmaker is a logical one. Even if we do not have an answer to what makes negative truths true, it is still logical possible for these truths to have a truthmaker. He (2019, 42) then claims that the same goes for "specific and general, concrete and abstract,

contingent and necessary, analytic and synthetic cases," and concludes PT is inductively justifiable.

With these principles in place, Jago sets out his argument for maximalism on the assumption for reductio that there is a truthmakerless truth, $p \wedge \neg TMp$. By PT it is logically possible for any truth to have a truthmaker and we have $TM(p \wedge \neg TMp)$ similar to step (5) in the Fitch argument. We should pay attention to the reasoning here, "[t]hat supposed possibility is a situation in which there is a truthmaker for: A and A has no truthmaker." (Jago 2019, 42) Jago picks out a possible world where $TM(p \wedge \neg TMp)$ holds in that world instead of $\Diamond TM(p \wedge \neg TMp)$. The reasoning here is not purely based on the axiom system but involves semantics. More on this in the following, now let us turn to Trueman's objection.

Trueman objects to Jago's premise outrightly. He thinks there is truthmakerless truth and this response has been anticipated as question-begging by Jago. Trueman goes on to devise another version of Fitch-style argument that leads to the conclusion that all truths are truthmakerless, and intends to show that the outcome of Fitch-style argument could also undermine maximalism. Trueman(2020, 4) uses a truthmakerless operator "L", which abbreviates for "… is true but it is not the case that it has a truthmaker." With this operator, he gets the following principles:

Possibility of truthmakerless (PL) $(A \wedge B) \rightarrow \Diamond(LA \wedge LB)$[4]

Factivity of truthmakerless (FL) $\Box(LA \rightarrow A)$

PL runs with Jago's inductive reasoning for PT, for every truth it is logically possible that it has no truthmaker. This principle spares the need for L distribution and could start the Fitch-style argument by taking $LA \wedge L\neg LA$ as assumption. FL follows from the definition of L operator. With these two principles, Trueman is able to derive a similar conclusion that if p is true then it is truthmakerless, i.e. $p \rightarrow Lp$. The weak logical possibility works for PT also excludes counter-examples to PL, it is at least logical consistent to have a possible world where the counter-example vanishes. Trueman makes the debate into a stalemate: the Fitch-style argument could get both truthmaker maximalism and truthmaker nihilism off the ground. Trueman (2020, 5) concludes "[n]one of the arguments we have reviewed in this paper is any more dialectically effective than any of the others."

[4] I want to point out that the PL principle in Trueman's argument, though able to generate the desired outcome, is different from PT since $(\Diamond LA \wedge \Diamond LB) \rightarrow \Diamond(LA \wedge LB)$ is not valid even in S5 system. The possibility of an atom formula being truthmakerless does not guarantee the possibility of two atom formulas being truthmakerless at the same world.

Jago acknowledges the force of Trueman's objection, and dials his maximalism argument back a little. Rather than convincing those who do not believe truthmaking, Jago sets to provide a Fitch-style argument for those who accepting truthmaking but not accepting maximalism due to the problem caused by negative truths. He replaces the truthmaking operator with the necessitating operator "Nec", the minimal requirement of truthmaking that some entity necessitates the truth. Jago(2020, 2) claims that Trueman's PL has a counterexample under Nec reading, $A \rightarrow \Diamond(A \wedge \neg NecA)$ is false when A is a necessary proposition such as $p \vee \neg p$ that makes $Nec(p \vee \neg p)$ "logical necessary." Given the Nec reading of truthmaking, PL faces a counterexample while PT does not. Jago's argument for maximalism becomes a conditional one: if you accept truthmaking, then you should embrace the Fitch-style argument for maximalism.

4.Logical necessity

Does Jago's counterexample refute the Trueman's Fitch argument? Obviously, $p \vee \neg p$ is a necessary truth in propositional modal logic and blocks Trueman's inference because the formula $(p \vee \neg p) \rightarrow \Diamond((p \vee \neg p) \wedge \neg Nec(p \vee \neg p))$ is false. However, the Nec operator is an operator with quantificational implications, a truthmaker should exist for $p \vee \neg p$. Why must there exist something as the truthmaker of necessary truth rather than nothing?

Consider using a model with an empty domain where the existential formulas are all false as in inclusive logic (Nolt 2020), Jago's defense becomes ineffective and the Trueman's Fitch gets an upper hand. All universally quantified formulas are true in the empty domain while nothing exists to serve as the truthmaker. Jago could respond that the introduction of empty domain deviates from the conditional strategy, by taking Nec one should already commit to the truthmaking theory and thus rejects the case where there are truths without truthmaker. But there is a complication caused by negative truths, to solve this some truthmaking theorists reject maximalism or necessitarianism. Moreover, the empty domain objection is not an ad hoc case that comes from the point of inclusive logic alone, there is some metaphysical motivation behind it (Saenz 2014, 92-93). Jago's Fitch is not favored even if one has already committed to truthmaking.

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