Cian Dorr, "Quantifier Variance and Collapse Theorems", section 1-4

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1. Quantifier Variance

(QV) There are different candidate meanings for quantifiers.

For (QV) to be philosophically interesting, the candidate meanings must be *quantifier* meanings: they must *behave like quantifiers* in inferences.

QV is often used to motivate metaontological deflationism: apparently different ontological views are "saying the same thing" in different ways. That requires all relevant parties using *quantifiers* in their language.

*This is essentially saying that they follow the same set of inference rules. Rules do not fully determine meaning.

2. Collapse Theorems of $\vee$

Collapse theorems claim that inference rules do determine meaning (up to mutual entailment). Dorr ilustrates this by disjunction.

Theorem 1: We may prove a collapse theorem with respect to an uninterpreted language S: let $\vee_1$ and $\vee_2$ be two syntactic objects that follow the usual inference and elimination rules of $\vee$, then:

(1) $\phi_1 \vee_1 \phi_2 \vdash \phi_1 \vee_2 \phi_2$

(2) $\phi_1 \vee_2 \phi_2 \vdash \phi_1 \vee_1 \phi_2$

Proof for (1). $\phi_1 \vdash \phi_1 \vee_2 \phi_2$, $\phi_2 \vdash \phi_1 \vee_2 \phi_2$, Therefore, by $\vee_{Elim}$ of $\vee_1$, $phi_1 \vee_1 \phi_2 \vdash \phi_1 \vee_2 \phi_2$ Proof for (2) is symmetric to (1).

This shows that there is at most one symbol playing the inferential role of $\vee$ in a syntax. Now let there be two languages L_1 and L_2. By Theorem 1, each language contains just one symbol for $\vee$.

The syntactic proof above won't help block the possibility of semantic variation, because syntactic rules cannot be applied *across* languages.

But this is not yet an argument for the claim that "$\vee$" has the same *meaning* in languages L_1 and L_2, both with the same syntax! The obstacle is that we cannot directly use inference rules on a formula in another language.

When ϕ_1 and ϕ_2 are formulas in L_1, the only relevant inferential rule is $\vee_1$-intro, so we cannot get any meaningful claim about $\vee_2$.

Local vs. Global rule-following: to address the difficulty we need to define rule-following on the level of propositions . The "local" definition (p. 509) captures the inferential behavior or $\vee$ on propositions that are expressible in the relevant languages. The "global" definition (p. 511) defines the inferential pattern on all propositions.

Propositions are the *semantic values* of syntactic structures/sentences, so they transcend specific languages.

Theorem 2: If L_1 and L_2 have the same expressive power (up to mutual entailment), then for any sentences ϕ_1, ψ_1 in L_1 and ϕ_2, ψ_2 in L_2, if ϕ_1 is equivalent to ψ_1 and ϕ_2 is equivalent to ψ_2 then $\phi_1 \vee_1 \psi_1$ is equivalent to $\phi_2 \vee_2 \psi_2$.

The intuition of this "local" version of Collapse is that with the help of expressive equivalence, we can *"translate"* between equivalent sentences in the two languages, and then run the proof for (1) after proper translation.

Theorem 3: If $F(p,q)$ and $G(p,q)$ are both the least upper bound of propositions p and q, then $F(p,q) = G(p,q)$

The intuition for the "global" version: regardless of whether a proposition is expressed (and expressed in whatever form) in a syntax, we may directly define inferential rules on propositions, and those rules determine uniquely the meaning of "disjunction".

We may use Theorem 3 to argue that the meaning of $\vee$ does not vary across languages.

Possible response: Dorr mentions that the only way to block Theorem 3 is to argue that the notion of entailment varies across languages (which means that the "Global" inferential properties we defined are in fact not univocal.)

Here is what I take to be going on. Let "entail" pick out relation R_1 in language L_1 but R_2 in L_2, then when L_1 speakers say "$F(p,q)$ entails r" they mean R_1 while L_2 speakers will mean R_2. Now, we may rigidify "entail" to mean whatever we mean, say R_2, and check whether their quantifiers satisfy the relevant "global" inferential properties defined in terms of R_2. But then whether the L_1 speakers take a sentence to be entailed by some other sentence provides *no evidence* whether that sentence is "really" entailed, in terms of R_2 by the other sentence.

[A section of "Tarskian" variations will be attached at the end. Dorr takes it to be a distraction.]

3. *Collapse Theorems of Quantifiers on Closed Sentences*

Define the entailment relation on the semantic values of closed sentences, and we may have the "local" (p. 522) vs. "global" (p. 529) versions of the inference rules concerning $\exists$. As before, local inference rules are defined on propositions expressed by sentences, and global inference rules are defined directly on all propositions.

3.1 The local version of collapse

Local inference rules are like the regular inference rules in logic textbooks (p. 522)

Theorem 4: If L_1 and L_2 has a name-mapping such that for every sentence ϕ_1 in L_1 there is a sentence ϕ_2 in L_2 equivalent to ϕ_1, and the names in ϕ_2 are the images of names in ϕ_1 via the mapping and vice versa, then if ϕ_1 in L_1 is equivalent with ϕ_2 in L_2, their existential closure are equivalent too.

The intuition for name mapping: we can translate between L_1 and L_2, and the translation preserves names.

Proof. Suppose ϕ_1 is equivalent to ϕ_2, by the local introduction rule of $\exists_1$, $\phi_1 \models \exists_1 x_1 \phi_1^*$. by equivalence between L_1 and L_2 there is a sentence ψ in L_2 that is equivalent to $\exists_1 x_1 \phi_1^*$, and ψ does not contain the image of the relevant constant that was in the place of x_1 in ϕ_1. Now, because ϕ_2 and ϕ_1 are equivalent, $\phi_2 \models \psi$, and ψ does not contain the relevant "image" constant. Then by elimination rule of $\exists_2$, $\exists_2 x_2 \phi_2^* \models \psi$, and ψ is equivalent to $\exists_1 x_1 \phi_1^*$ The converse is parallel.

Two problems about Theorem 4: First, it relies on name mapping but quantifier variantists may claim that different languages have different stocks of names (those who apparently quantify over more things will also have more names in their language). Second, it is not clear that our language satisfies the local introduction and elimination rules in full generality.

Problems with local introduction of $\exists$ (A) empty names (if a is empty then $\phi(a)$ does not entail $\exists x\phi(x)$). (B)) contingentism ($\forall x(x \neq a)$ is contingently false, but $\exists y\forall x(x \neq y)$ is necessarily false. So $\exists$-elimination makes a contingently false proposition entail a necessarily false proposition, but this cannot hold if entailment is metaphysical necessitation).

The best shot for Theorem 4: we apply Theorem 4 not to $\exists$ but to $\exists_\diamond$ where $\exists_\diamond$ is the possibilist quantifier, this will block problem B. Moreover , for the quantifier variantists, difference in the meaning of $\exists$ will typically result in difference in the meaning of $\exists_\diamond$, but Theorem 4 will preclude any meaning variation on $\exists_\diamond$.

The intuition: the possibilist existential quantifier quantifies over all possibilia including those that are not actual.

The example of being huge on p. 526: let "something is huge" be false under one quantifier meaning but true under another quantifier meaning, then "possibly something is huge" could be false under one quantifier meaning but true under another meaning

Problems with local elimination of $\exists$ If entailment is metaphysical necessitation, then "Hesphorus is a gas giant" enatils "Phosphorus is a gas giant" but $\exists x G(x)$ does not. .

Note that in "Phosphorus is a gas giant" there is no occurrence of "Hesphorus", so we may run $\exists$-Elim.

Dilemma: To save $\exists$-Elim we may consider a super fine-grained notion of propositions and a Tarskian notion of entailment under which "Hesphorus is a gas giant" does not entail "Phosphorus is a gas giant" (because there are Tarskian permutations that changes the reference of "Phosphorus" but not "Hesphorus"). This move blocks the counterexample. But simultaneously it makes Theorem 4 *useless* because in this super-fine-grained conception the idea of expressive equivalence is super demanding, and it is unlikely that quantifier variantists will endorse it.

3.2 *The global version of collapse*

In the global version of collapse we must define the inferential rules of quantifiers directly on the level of propositions/semantic values. The natural thought is that quantifiers are functions from concepts (the semantic values of predicates) to propositions.

Quantifiers as functions are defined on the space of all concepts, but the inference rules are only applicable to the concepts that are not about the specific objects already mentioned in proofs/undischarged assumptions]

Global $\exists$-Intro: If proposition p predicates concept c of some object and c is not about that object, $p \models F(c)$

Global $\exists$-Elim: If proposition p predicates a concept c of some object and entails some proposition that is not about that object then $F(c) \models q$

Theorem 5: If F and G both follow the Global inference rules, and c is a concept not about every object, then $F(c)$ and $G(c)$ are equivalent.

Proof. Let x be some object that c is not about, and let q be the proposition that attributes c to x. By Introduction, q entails G(c), and it is not about x. q is the result of removing F from F(c), so by elimination rule F(c) entails G(c). The converse is parallel.

Objection 1 (Sider): Theorem 5 assumes that there is a common stock of concepts across languages, but that is implausible. Apparently different ontological views will result in different stocks of concepts and objects across languages.

Response (Dorr): This is radical, and also unmotivated, because it undercuts the motivation for QV–why not attribute meaning variations to predicates?

The obvious response from Sider or a quantifier variantist is that differences in predicate meaning are *explained* by differences in quantifier meaning

Objection 2 (Dorr): Even if we grant that all languages have a common stock of concepts, the quantifier variantist can still deny that the quantifiers all have the global inferential properties. Note that the properties are themselves defined by *quantification* over objects, but it is question-begging to assume that all languages quantify over objects in the same way. If each language has its own quantifier meaning, then in each language the clauses for ∃-Elim and ∃-Intro will pick out different properties too.

A simplistic way to think about this: Theorem 5 not only assumes a common stock of concepts but also a common stock of objects. But if QV is true then the notion of objecthood must be revised too.

Dorr's response carries over to "Tarskian" variations of the Collapse argument if the notion of a legitimate variant involves quantification over objects. And it is not clear how to define Tarskian variants otherwise.

4. Collapse Theorems of ∃ in Open Sentences

Tentative thought: open sentences express propositions relative to variable assignments, so we may define inferential rules relative to variable assignments.

Problem: Analogous to Dorr's Objection 2 to Theorem 5, there is no guarantee that different quantified languages will quantify over variable assignments in the same way, so the definition of the relevant inferential properties may fail to be univocal across languages.

Solution (of Dorr): take open sentences to be something like predicates or concepts, and directly define entailment relations on concepts. So, "Red(x)" *is* "being red". We then define entailment on the space of all concepts.

To make a dialectical progress the new definition of entailment must not involve any quantification over objects: we cannot say concept *c* entails concept *d* if everything falls under *c* falls under *d*. Dorr's alternative definitions are:

(1. Entailment via concept conjunction) c entails $d =_{df}$ c is the conjunction of c and d.

(2. Entailment via Identification) c entails $d =_{df}$ To be c is to be c and to be d

Think of it in terms of determinate/determinables: let c be *being crimson* and let d be *being red*, then 1 says "x is crimson" entails "x is red" just in case "x is crimson" says the same thing as "x is crimson and red." 2 is an alternative formulation of the same point.

Problem: when we go beyond monadic concepts the use of variables is indispensable: we need to (at least be able to) distinguish a relation from its converse.

Solution: An open sentence expresses a n-adic concept relative to a non-repeating sequence σ of n variables. Notation: ${}^{\sigma}[[\phi]]_L$

Sequence-theoretic Consequence: $\phi \vdash_L \psi =_{df} {}^{\sigma}[[\phi]]_L$ entails ${}^{\sigma}[[\psi]]_L$ for every σ that covers both ϕ and ψ

"Covers" just means that σ contains all variables free in some open sentence.

This definition does not assume objectual quantification because the variables in the sequence are not assigned to any value. They just distinguish the argument slots in a relation.

4.1 The local version of collapse on open sentences

Local ∃-Intro: (Q has this property iff) for any ϕ, ν and σ covering ϕ, ψ, ${}^{\sigma}[[\phi]]_L$ entails ${}^{\sigma}[[Q\nu\phi]]_L$

Local ∃-Elim: (Q has this property iff) for any ϕ, ψ, ν and σ covering ϕ, ψ, if ${}^{\sigma}[[\phi]]_L$ entails ${}^{\sigma}[[\psi]]_L$, then ${}^{\sigma}[[Q\nu\phi]]_L$ entails ${}^{\sigma}[[\psi]]_L$.

Revisit the old problems: For local intro rule, the problem is that the concept of *not identical to anything* would have to entail *being such that something is not identical to anything*, but the application of the former to some given object is contingently false while the application of the latter is necessarily false. Dorr takes this to be solved in the same way as in the case of theorem 4 (see above): we apply the rules not to $\exists$ but to $\exists_{\diamond}$

For local elimination rule, the problem concerns essentialist truths, but such cases won't arise if we operate with open sentences/concepts.

The old problem: "David Lewis is either a poached egg or a galactic emperor" entails "someone is a galactic emperor" because David Lewis couldn't have been a poached egg, but "Something is either a poached egg or a galactic emperor" does not, violating the elimination rule. No counterpart counterexample arises in the case of open sentences for *being a poached egg or a galactic emperor* does not entail *being a galactic emperor*

Constraints on "well-behaved" languages To get collapse theorems on languages L_1 and L_2 we must assume that they are well-behaved, which means that they satisfy seven conditions:

Converse constraint: changing the order of variables in a sequence is equivalent to generating a generalized converse of the concept (relation).

$\sigma \circ \pi$ is to let π go into σ, so ${}^{\sigma\circ\pi}[[\phi]]$ is to change the variable-order in σ by the order of subscripts in π

Expansion constraint: adding an empty argument place is equivalent to applying a longer sequence of variables to a concept.

Alphabetic variation: *which* variables are used in the sequence won't matter to the identity of the concept.

Combining converses: $Conv_{\pi_1} \circ Conv_{\pi_2}$ (c) = $Conv_{\pi_1 \circ \pi_2}$ (c) (Changes in the order of variables can be combined)

Trivial converse: if π is identity permutation then $Conv_\pi(c) = c$

Converse entailment: c entails d iff the converse of c under permutation π entails the converse of d under π

Necessary connection between a relation and its converse

Expansion entailment: c entails d iff the expansion of c entails the expansion of d

Adding abundant argument places does not matter to entailment patterns.

Constraint of predicative equivalence: for every formula ϕ_1 in L_1 and sequence σ_1 there is a formula ϕ_2 of L_2 and a sequence σ_2 such that ${}^{\sigma_1}[[\phi_1]]_{L_1} = {}^{\sigma_2}[[\phi_2]]_{L_2}$

Theorem 6: Suppose Q_1 and Q_2 have the "local" properties, and L_1 and L_2 are predicatively equivalent and well-behaved, then if ϕ_1 and ϕ_2 are equivalent, $Qv_1\phi_1$ and $Qv_2\phi_2$ are equivalent too.

Somewhat sloppy for I dropped all the brackets

Proof in Appendix A

4.2 Global version of Collapse on open sentences

The intuition here is that the semantic "contribution" of $\exists$ is to change the concept *being bright* to the proposition *that something is bright*.

Compositionality: ${}^{\sigma}[[Qv\phi]]_L = [[Q]]_L({}^{\sigma,v}[[\phi]]_L)$

Global $\exists$-Intro: (F has this property iff) c entails $Exp(F(c))$

Appendix B shows how Compositionality and the global rules jointly entail the local rules.

Global $\exists$-Elim: (F has this property iff) whenever c entails $Exp(d)$, $F(c)$ entails d.

Theorem 7: Suppose F and G both have the global properties, then F(c) and G(c) are equivalent for any concept c of positive addicity.

Proof. Since G obeys the Intro-rule, c entails $Exp(G(c))$, so $F(c)$ entails $G(c)$ by the elimination rule of F

By defining quantifier meanings and entailment relations directly in the space of concepts we avoid any problematic use of names (which blocks the local version of collapse on closed sentences) or quantification over objects (which blocks the global version of collapse on closed sentences.)

5. Philosophical assessments

Dorr considers the best response of the quantifier variantist to be denying that quantifiers in all relevant languages have the (local or global) inferential properties. So, we consider some developments of this view.

Note that the speaker of each language will formulate the clauses that define those properties *in their own language*. Define a language to be self-vindicating if its speakers can *truthfully* say "the quantifier in my language obey the (Intro or Elim) rules defined as such ..."

5.1 All self-vindication

Possibility 1: all languages are selfovindicating. Their speakers all speak the truth when they say that their quantifiers follow certain rules, but in fact their definitions of those rules pick out different inferential properties, so it is not true that the quantifiers of all languages share enough inferential properties that give rise to Theorem 6 or 7.

A demonstration: take the concept *being a chair*, the universalist takes this concept to entail Exp(*something is a chair*) , but the nihilist or the organicist will claim that "something is a chair" is false, and its expansion must be empty (if P is false then nothing is such that P). So, from the universalist's perspective, the nihilist language must lack Global ∃-Intro.

Because the universalist thinks *being a chair* is not empty

Symmetrically, the universalist's language will lack Global ∃-Elim from the nihilist's perspective.

Given that in all the languages the properties are defined in the same way, where does the equivocation come from?

Option A: Different languages mean different things by "entail".

Think of it this way: There is a common stock of concepts but there is an abundance of entailment-like relations.

Option B: Different languages have different stocks of concepts, when universalists uses "being a chair" to pick out a concept c_u that concept is different from c_n picked out by the nihilist.

Each language has its own stock of concepts, but quantifiers are defined in the usual way on each stock of concepts.

Dorr's response to option B: "the whole point of bringing in concepts was to provide a neutral framework for characterising the semantic contributions of predicates in languages whose quantifiers may be different from ours; but if concept-talk is highly idiosyncratic to us, it seems misguided to assume that the predicates of the other languages express concepts at all."

So Dorr thinks the best response is to say that entailment works differently in each language.

5.2 No self-vindication

If no language is self-vindicating, then even in our own language entailment cannot be defined as "for any object x if it falls under c it falls under d"–this is not even *true*. But maybe there is a super-fine-grained notion of concept and entailment in which that is not true. We might work on that sense of entailment ...

5.3 Some but not all languages self-vindicate

This could be a reason to choose a "joint-carving" quantifier. But this is not agreeable in spirit to quantifier variance which is usually taken to be a deflationary thesis.

6. Warren's response to Dorr

I want to put Warren's response in very simplistic terms because it does not hinge on any techinical details of Dorr's theorem 6 or 7. For I take Warren to be essentially employing a *metasemantic* response to Warren, and it goes roughly like this:

1. Take two communities who are actually using their languages in the universalist/nihilist ways. There is a strong metasemantic pressure to interpret them as both speaking the truth. A

2. Quantifier variance is the consequence of this (correct) metasemantic picture.

3. The formal results of theorems 6 or 7 cannot block this metasemantic argument.

4. So the formal results is dialectically effective against quantfieir variance.

I take these points to be what Warren is gesturing at when he emphasizes that quantifier variance is the result of the "use" of the quantifiers.

Quantifier variance/quantifier elimination

Another theme in the Warren response is that quantifier variance can be reformulated in terms of quantifier elimination (this is anticipated by Hirsch) or concept elimination.

1. Suppose we take option B response to Dorr's theorems 6 and 7. Then there is some sense in which not all language speakers are using *concepts*. (For we may rigidify "concept" to mean whatever *we* mean by "concept".) But the quantifier variantist can happily accept this. Warren says:

"Concepts" with a big C

"Surely when looking at the intricate ways that speakers of an alien language use expressions that function syntactically like predicates, I can reasonably surmise that said expressions are meaningful in a general sense, even without having to think they express concepts that I can already express with my own predicates. We can put this by saying that while they don't express concepts in our sense, they do express something like concepts. So they may be "meaningless" in a strict sense, but are meaningful in a broader sense." (p. 751)

2. More generally, it might be argued that a term *Q* deserves to be called a "quantifier" only if it follows the relevant (local or global) rules. So the quantifier-like term in some other language won't deserve to be called "quantifier" in the sense specified by one's own language because each language picks out its own inferential property using the rules.

"Quantifiers"–with a big Q

Dorr considers this to be chauvinstic and objectionable. But Warren will be once again happy accepting this proposal–what he calls "quantifier elimination"–and he points out that this is already anticipated in Hirsch (2002):

"My response to Dorr's argument admits that, in a rich, semantic sense, each language might view only its own quantifiers as genuine (in this case, as obeying the D-rules as expressed in their language).[...construes]This is true, but let us remind ourselves that quantifier variance is a doctrine concerning only "quantifiers" construed as expressions obeying certain formal-syntactic rules in various possible languages. As soon as claims about "quantifiers" are understood as requiring more than this, they cease being relevant to quantifier variance. [...] In fact, Hirsch stressed this point long before the appearance of Dorr's paper" (p. 752)

The thought of Hirsch is that the intuition for quantifier variance is stable even if one may formulate the thesis in different ways: it is no less puzzling (or deflationary) if there is another community that can describe the world equally well without using a quantifier (or notions like existence, etc)–for that would just show quantifiers (or ontology) is not indispensable or stable across languages.