Andrew Bacon 2019

Christopher Sun

I henceforth refer to the "After Physics: The First Philosophy" handout as APTFP. Please first see APTFP §3.1 for the idea that QUALITATIVISM and NIHILISM, FUNDAMENTAL QUALITATIVISM and FUNDAMENTAL NIHILISM stand or fall together.

MOTIVATIONS: Physical theories can be elegantly formulated *geometrically* upon the class of physically possible qualitative states (e.g., symplectic manifold in classical mechanics, Hilbert space in quantum mechancis). In this formalism, possible worlds do not need to have any internal structure (i.e., individuals standing in relations). Possible worlds can just be primitive points standing in primitive relations.

This is also Structuralism, albeit it is primarily motivated by economy/parsimony rather than arguments from symmetry. Therefore, Bacon and Dorr's anti-individual Structuralism is *different* from Dasgupta's flavour. The most important distinction lies in that Bacon does not *eliminate* possibilities related by symmetry operation, but "*recreates* every physical possibility an individualistic metaphysics postulates".

1 Finean Structures

Let D^e be any set and D^t be any complete and atomic Boolean algebra. Let M be the permutation group of D^e and suppose we also have an action of M on D^t.

(Note that if we assume not only Booleanism but Intensionalism then $D^t = \wp W$ for the set of possible worlds W. If we furthermore have an action of M on W, than we can lift it to an action of M on D^t: $\pi p = \{\pi w \mid w \in p\}$, for any $\pi \in M$.)

These actions on D^e and D^t determine a unique Substitution Structure where the action on $D^{\sigma\to\tau}$ is defined by: $\pi f = \pi \circ f \circ \pi^{-1}$, for any $f \in D^{\sigma\to\tau}$ and any $\pi \in M$.

- a is *metaphysically definable* from X iff every $\pi \in M$ that fixes X fixes a.

(Note: this is a substitution-theoretic characterization of metaphysical definition. We can also understand it *syntactically* as $\mathrm{MD}(y, \bar{x}) := \exists X(\mathrm{Pure}(X) \wedge y = X\bar{x})$.)

- p is *about* $a_1 ... a_n$ iff p is metaphysically definable from $a_1 ... a_n$ and p is not metaphysically definable from any proper subset of $a_1 ... a_n$.
- p is *qualitative* iff p is not about anything iff p is metaphysically definable from $\varnothing$ (that is, p is pure) iff $\pi p = p$ for every $\pi \in M$.

2 Fundamental and Non-fundamental

QUALITATIVE REALITY: $\bigwedge_\sigma \forall x^\sigma(\mathrm{Fun}^\sigma(x) \to \mathrm{Qual}^\sigma(x))$.
This entails FUNDAMENTAL QUALITATIVISM and FUNDAMENTAL NIHILISM.

It is easy to show that Closure of Qualitativeness holds. But then to avoid FUNDAMENTAL QUALITATIVISM collapsing into QUALITATIVISM, we must reject FUNDAMENTAL COMPLETENESS as fleshed out using *metaphysical definition*. (APTFP §2.3.) As a Booleanist, we may nonetheless preserve FUNDAMENTAL COMPLETENESS by fleshing it out using good old *modality*. Supervenience thesis (type t):

- Every truth is metaphysically necessitated by some truth that can be metaphysically defined from fundamental elements.

Assume that propositions form a complete and atomic Boolean algebra, then every proposition is isomorphic with the set of maximally strong consistent propositions (*world propositions*) that entail it. Since qualitativeness is closed under Boolean operations, the qualitative propositions form a complete and atomic subalgebra. Then it follows from the Supervenience thesis that *every proposition is necessarily equivalent to a qualitative proposition*. We have a hyperintensional theory.

To make Fundamental Completeness (Supervenience) true, fundamental elements must be able to give a complete account of the space of possibilities. That is, we must be able to metaphysically define enough propositions from fundamental elements to uniquely specify (necessitate) each metaphysically possible world.

3 Populating D^t

We can metaphysically define a proposition from a fundamental predicate (e.g., type $e \to t$) and a fundamental predicate functor (e.g., type $(e \to t) \to t$).
(This functorese strategy is pursued by Shamik Dasgupta and Jason Turner.)

For Nihilism, however, this strategy has the following two vices:
(i) The functorese replacement for singular quantification may not be ontologically innocent, since it seems to be different from an orthodox Fregean account of quantification only in variable-binding, but not in treatment of quantifier proper.

(ii) According to certain functionality principles, differences between elements of type $\sigma \to \tau$ should be grounded in differences between their behaviors (combining with elements of type σ to yield elements of type τ). Since $D^e = \varnothing$ for Nihilists, there are not enough predicates and functors with different behaviors to generate all the different propositions needed to respect Fundamental Completeness.

Objection: But Nihilists should not accept functionality principles in the first place! For Nihilists, differences between predicates should be *primitive* and certainly not pinned down by their behaviors on type e (individuals). More generally, functionality might just be one way to make precise the intuition that "relata are prior to relations that relate them", but any true structuralist shall certainly reject this idea!

Given functionality principles, the only way left for Nihilists to populate D^t is by postulating fundamental elements of *hereditarily propositional types* (pure types). This is called Fundamental Propositionalism (Bacon) or Priorian Nihilism (Dorr).

4 Priorian Nihilism

A naive way to achieve this is to postulate a fundamental proposition for each metaphysical possible world. Since $D^e = \varnothing$, these propositions are all qualitative. But (i) this is not parsimonious; (ii) this violates Fundamental Independence; (iii) this makes it unlikely to formulate simple laws of fundamental physics.

What, then, should we postulate as fundamental elements in order to respect Fundamental Completeness (Supervenience)? It is easy to see that this depends on what our modal space is like. We shall first take classical mechanics as a concrete example, and generalize to a broad class of individualistic physical theories later.

In Hamiltonian mechanics, we have a symplectic manifold $\langle \mathcal{M}, \omega^2 \rangle$ where $\mathcal{M} = \mathbb{R}^{6n} = \{\langle \mathbf{p}_1, \ldots, \mathbf{p}_n, \mathbf{q}_1, \ldots, \mathbf{q}_n \rangle\}$ and $\omega^2 = \sum_{i=1}^{n}(d\mathbf{p}_i \wedge d\mathbf{q}_i)$, and a scalar field H over $\mathcal{M}$ (the *Hamiltonian*). We can arrive at Hamilton's canonical equations by taking the Hamiltonian vector field generated by H as the phase velocity.

(Canonical Equations: $\frac{d\mathbf{p}_i}{dt} = -\frac{\partial H}{\partial \mathbf{q}_i}, \quad \frac{d\mathbf{q}_i}{dt} = \frac{\partial H}{\partial \mathbf{p}_i}$)

A possible state in the state-space of a system will evolve towards a direction perpendicular to the direction in which the total energy of the system is decreasing.

Possible states are maximally strong consistent propositions (those atoms of the Boolean algebra that are metaphysically possible). They could not all be fundamental. Instead, we postulate the following fundamental elements to define them:

- Betweenness (a 3-place operator of world propositions, type $t \to t \to t \to t$):
  Bet $(p, q, r) = \top$ iff q lies on a *straight* line in the state-sapce between p and r,
  Bet $(p, q, r) = \bot$ iff otherwise. This gives the *affine* structure of the modal space.

- Congruence (a 4-place operator of world propositions, type $t \to t \to t \to t \to t$):
  Cong $(p, q, r, s) = \top$ iff the *distance* between p and q is the same as the *distance* between r and s, Cong $(p, q, r, s) = \bot$ iff otherwise. This gives the *metric* structure of the modal space. We formulate Canonical Equations using Bet and Cong.

- Almost-Sameness (a 1-place particle operator of world propositions, type $t \to t$): $[\mathrm{a}]p$ is entailed by a possible world s iff p is entailed by all possible worlds that agree with s concerning the positions and momenta of all particles apart from a.[1] These operators satisfy S5 logic each and multi-modal product logic together.

- Particle-Foliation (an operator functor of operators, type $(t \to t) \to (t \to t)$): $\square^* p$ is entailed by a possible world s iff p is entailed by all possible worlds that stand in the equivalence relation with s, such that each equivalence class of this relation is orthogonal to every equivalence class of the equivalence relation of $\square$.

- Particle-State Proposition (a proposition, type t): A^p (A^x) is entailed by a possible world s iff some particle in s has momentum p (location x).

$[\mathrm{a}]^* A^p$ is entailed by a possible world s iff the particle a has momentum p in s. Since every possible world in classical mechanics can be completely specified by stating the location and momentum of every particle, we now have enough fundamental elements to make Fundamental Completeness (Supervenience) true.

These fundamental elements are of hereditarily propositional types and are of course qualitative, since $D^e = \varnothing$. Moreover, they (i) are parsimonious; (ii) does not violate Fundamental Independence more than the individualistic counterpart does; (iii) can formulate Hamilton's Canonical Equations in a relatively simple manner.

WOOHOO!

It is now easy to generalize this procedure to any individualistic physical theory where each possible world can be completely specified by stating *the distribution of a collection of fundamental properties and relations of individuals of any arity*.

I leave out certain important issues in this 2019 paper, but the night is darkening and Andrew Bacon's writing is as profound and intense as the deep ocean.

[1] This completely determines what proposition $[\mathrm{a}]p$ is, since every proposition in our Boolean algebra is isomorphic with the set of world propositions that entail it.

5 Haecceitism?

Let me conclude with a remark on the relation between Bacon and Dorr's Nihilist view and Dasgupta's structuralist Generalism motivated by symmetry arguments:

There could be non-trivial permutations of the modal space that preserve the geometric relations while mapping the true point to a false point. These correspond to permutations of individuals figured in symmetry arguments. Hence Nihilism is compatible with Haecceitism. Indeed, if we eliminate haecceitistic possible worlds, then we are left with a quotient space the structure of which is hard to capture intrinsically and in which it is not obvious how to formulate laws. Hence Nihilism is not friendly to Anti-Haecceitism and thus any structuralist thesis that entails it.

Furthermore, the mere coherence of Haecceitistic Nihilism shows that symmetry arguments might be targeting the wrong place. See Bacon's concluding paragraph:

> It is not at all obvious that individuals are responsible for invisible differences, if the invisible differences still arise in theories formulated without individuals. This raises an important moral: If we are worried about invisible differences we should be more concerned with the content (understood broadly) of the theories that generate them, such as classical mechanics, rather than the choice of whether to formulate those theories in terms of individuals or not.

Please do not circulate.
2021. 4. 4

STRUCTURA
Reading the Structure of the World
readingthestructure.weebly.com