Alexander Roberts 'From Physical to Metaphysical Necessity'

Tom Lingzhi Shi

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1 Physical Necessity

Nomological Bound (informal): Nothing is objectively possible beyond what is physically possible.
Nomological Bridges (informal): The boundaries of objective possibility can be characterized in terms of physical necessity.

Model – World^{+}: p is physically possible at a world just in case p is true according to some world proposition associated with a model of the laws of nature.
PN: A world v is physically possible from a world w if and only if every law of w is true at v.

PN*: A world v is physically possible from a world w if and only if every law of w is a law at v.
PN†: A world v is physically possible from a world w if and only if v has the exact same laws as w.

2 Formal Preliminaries

2.1 The core theory-Booleanism

PC: All instances of propositional tautologies.

MP: From A and $A \rightarrow B$ infer B.

Gen: From $A \rightarrow B$ infer $A \rightarrow \forall_\sigma x B$ when x does not occur free in A.

UI: $\forall_\sigma x A \rightarrow A[t/x]$(where $t : \sigma$ and is substitutable for x in ϕ).

$\beta\eta$: $A \leftrightarrow B$ whenever $A : t$ and $B : t$ are $\beta\eta$ equivalent.

RE: $A =_t B$, whenever $A \leftrightarrow B$ is provable from these axioms and rules.

According to Booleanism, there is a unique tautologous proposition.

2.2 Necessities

Definition 1 *(Weak Necessity)* $Nec^- := \lambda X.X\top$

Definition 2 *(Necessity)* $Nec := \lambda X \forall Y(Nec^-(Y) \rightarrow YX\top)$

Proposition 3 $L := \lambda p.(\top = p)$ *is a weak necessity.*

Proposition 4 *If X is a necessity, then* $X\top = \top$.

PROOF Since $Nec(X)$, by definition we have $\forall Y(Nec^-(Y) \rightarrow YX\top)$. It follows that $LX\top$, which is $X\top = \top$. □

Definition 5 *(Broadness)* $Br := \lambda Y \lambda Z \forall X(Nec(X) \rightarrow X\forall p(Yp \rightarrow Zp))$

2.3 Objective Necessities

Definition 6 *(Kripke Necessity)* $K := \lambda X(Nec(X) \wedge \forall Y(O(Y) \rightarrow Y \forall p \forall q(X(p \rightarrow q) \rightarrow Xp \rightarrow Xq)))$

$Kripke$ $\forall X(O(X) \rightarrow K(X))$

$Truth$ $O(\lambda p.p)$

$Basis$ $O(\blacksquare)$

$Composition$: $\forall X \forall Y(O(X) \wedge O(Y) \rightarrow O(\lambda p.XYp))$

$Conjunction$: $\forall X \forall Y(O(X) \wedge O(Y) \rightarrow O(\lambda p(Xp \wedge Yp)))$

Definition 7 *(Iteration)* $It := \lambda Y_1 \lambda Y_2 \forall X(X \lambda p.p \rightarrow (\forall Y_3(XY_3 \rightarrow X \lambda p.Y_1 Y_3 p) \rightarrow XY_2))$

Definition 8 *(Closure Operator)* $Cl := \lambda Y \lambda p.\forall X(It(Y)X \rightarrow Xp)$

$Closure$: $\forall X(O(X) \rightarrow O(Cl(X)))$

Let BO be the system which results from adding these principles to the core theory and closing it under *modus ponens*, universal generalization, and the modest rule that if $\phi \in BO$ then $\forall X(O(X) \rightarrow X\phi) \in BO$.

Definition 9 *(Objective Broadness)* $Br^O := \lambda Y \lambda Z \forall X(O(X) \rightarrow X \forall p(Yp \rightarrow Zp))$

Definition 10 *(O-broadest objective)* $BON^O := \lambda Y(O(Y) \wedge \forall X(O(X) \rightarrow Br^O(Y, X)))$

$Nomological\ Bound$ (formal) $BON^O(\blacksquare)$

Proposition 11 $\vdash_{BO} \neg \forall p(\blacksquare p \rightarrow \blacksquare\blacksquare p) \rightarrow \neg BON^O(\blacksquare)$

3 BOA

$Anti-Isolation$: $\forall X \forall Y(O(X) \wedge O(Y) \rightarrow Y\exists Z\forall p(It(\blacksquare)Z \wedge (Zp \rightarrow Xp)))$

Let BOA be the system which results from adding Anti-Isolation to BO and closing it under *modus ponens*, universal generalization, and the modest rule that if $\phi \in BO$ then $\forall X(O(X) \rightarrow X\phi) \in BO$.

3.1 Modal Subtraction Argument

There is some physically possible world in which one of the actual laws fails to be a law and, crucially, in which no other laws replace it. Therefore, a lawless world is reachable from the actual world by chains of physical possibility.

Consider an X-possible world w (X is an arbitrary objective necessity). Since from that lawless world, w is physically possible, w is also reachable from the actual world by chains of physical possibility.

Proposition 12 $\vdash_{BOA} BON^O(Cl(\blacksquare))$